Style Research Portfolio Analyzer

Guide to Performance Attribution Module

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A Guide to Style Research Performance Attribution

Introduction

The Style Research Performance Attribution Module is a holdings-based performance system, fully integrated within the Style Research Portfolio Analyzer (SRPA). It provides a variety of ways to assess sources of performance, including flexible Style definitions and user-controlled options to reflect the manager's investment process. It enables the separation of returns from asset allocation decisions implicit in country, sector or Style positions from those of stock-picking within those categories. Furthermore, the default hierarchical approach closely mirrors the standard SRPA risk decomposition, permitting the assessment of whether a manager's bets have paid off.

This document is intended to be a comprehensive guide to the *output* of the Style Research Performance Attribution Module. It assumes that the reader is already familiar with the process of generating such a report – please see the main help file (accessible via F1 in SRPA), if not. That said, in the course of this document there will be occasional references indicating how to tailor the analysis to provide alternative return breakdowns.

Organization of this document

There are a number of ready-to-print pages in the standard performance report. Using the example of a global portfolio case study, this guide walks through each of these reports, identifying and defining the various items. The main body of this document is a high level description of the key features and how to interpret them. More technical definitions may be found in Appendix P2. In addition, all of the data behind the charts (and much more besides) may be found in the data sheets towards the end of the performance output. Some of this is highlighted in the course of describing the standard pages. However, the majority of this data is relegated to Appendix P1, where all such sheets are covered comprehensively.

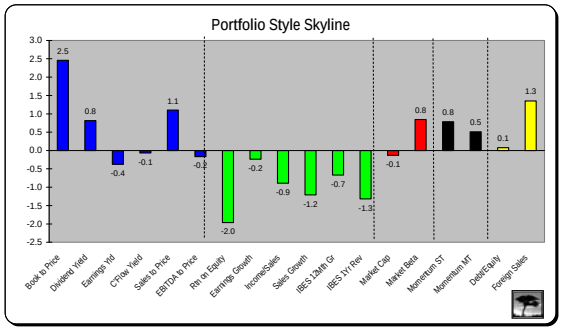
It is intended that the main body of this document will suffice for most reviewers of the performance reports. However, the more technically oriented performance analyst will find complete and precise definitions for all the performance data in the Appendixes.

The Case Study

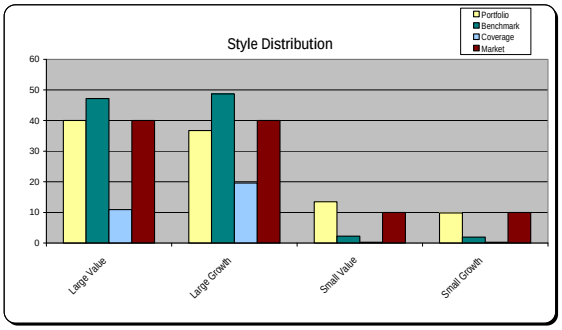
We focus on a portfolio invested in the Developed markets, analyzed over the period June 2004 through September 2005 and benchmarked against a World index. We show the '*Front Page*' of this portfolio as of June 2005 overleaf, in order to set the scene for our examination of its performance.

The Front Page – Case Study June 2005

A Value Portfolio



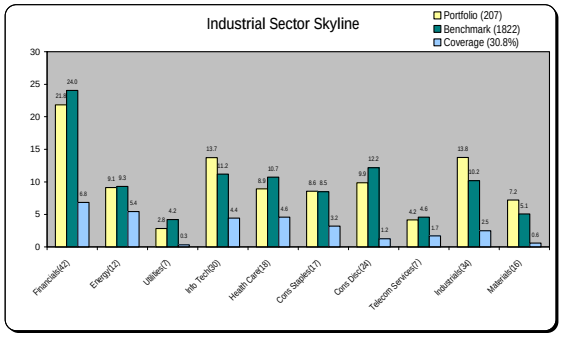
Style Skylines identify the major benchmark-relative Style tilts across the portfolio. Additional analysis factors and more detailed sector (and country) adjustment may also be informative.



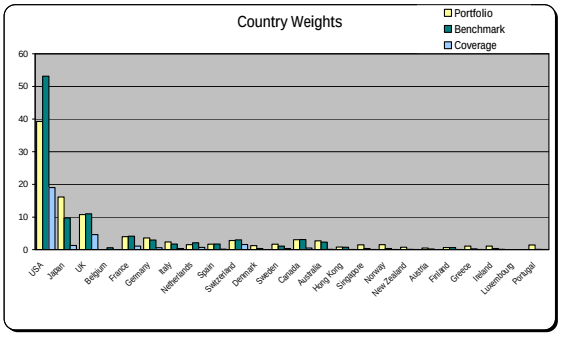
The Style Distribution shows the proportions of the portfolio and the benchmark invested in each of the basic Styles: Large, Small = Market Cap (80%, 20%) , Value, Growth = Book to Price per Share (50%, 50%). More detailed sector (and country) adjustment may also be informative.

Under-weight Large and over-weight Small

Sector bets within +/-4%



Sector Skylines show the sector weights within the portfolio and the benchmark. Coverage data reveal the diversification across the benchmark, for each sector and for the portfolio overall.



Country weights are represented across the portfolio and the benchmark. Coverage data reveal the diversification across the benchmark, for each country and for the portfolio overall.

Some sizeable country bets – overweight Japan, under-weight US

Moderate tracking error, from various sources, especially country risk

Risk Attribution

Coverage 30.8	Risk %		
Tracking Error	2.5		
Persistence Ratio	0.9		
Components of Tracking Variance		100%	
Currency Risk (Pure)	1.15	17.9	
Market X'terms	-0.53	-8.2	
Market Risk (Pure)	3.06	47.6	
Sector X'terms	-1.74	-27.0	
Sector Risk (Pure)	1.44	22.4	
Style X'terms	0.24	3.7	
Style Risk (Pure)	0.50	7.8	
Equity X'terms	0.05	0.7	
Equity Risk (Pure)	2.26	35.1	
Portfolio Beta	0.95	# Stocks	Actual / Effective
Portfolio Volatility	13.99	Port	207 / 152.0
Benchmark Volatility	14.52	Bench	1822 / 323.7
Correlation (Port, BM)	0.98		

⇦ Risk Attribution focuses on the estimation, analysis and decomposition of Tracking Error (and Tracking Variance) risk. The decomposition of the Tracking Variance identifies the principal sources of risk and further reveals risk compounding and risk offsets (via the Cross-Terms). The Persistence Ratio estimates the extent to which we can expect the Tracking Error estimate to be affected by any trending relative price behaviour of the stocks currently held in the portfolio.

Contribution to Tracking Variance by Style

Style	Total	Cur Risk	Market X' Terms	Market Risk	Sector X' Terms	Sector Risk	Style X' Terms	Style Risk	Equity X' Terms	Equity Risk
No Style Specified	0.00	0.00	0.00	0.01	0.00	0.00	0.00	0.00	0.00	0.00
Large Value	2.09	0.17	-0.04	1.00	-0.42	0.28	0.06	0.07	0.01	0.96
Small Value	0.21	0.28	-0.14	-0.78	0.23	0.25	-0.06	0.15	0.02	0.25
Large Growth	3.85	0.27	-0.18	3.02	-1.77	0.94	0.33	0.26	0.01	0.98
Small Growth	0.12	0.22	-0.13	-0.19	0.22	-0.03	-0.08	0.02	0.01	0.08
Cash holdings	0.17	0.22	-0.03	0.00	0.00	0.00	-0.01	0.00	0.00	0.00
Total	6.44	1.15	-0.53	3.06	-1.74	1.44	0.24	0.50	0.05	2.26

The Contribution to Tracking Variance (Risk) by Style identifies the sources of risk according to the Style of stocks carrying the risks and according to the level of contribution. The analysis highlights the Style(s) contributing the largest bets and further reveals how the risks are distributed at the overall Sector or Style levels and at the more individual stock specific level. The risk decomposition in the Equity Risk column identifies the Style categories in which the portfolio is taking its largest stock specific bets.

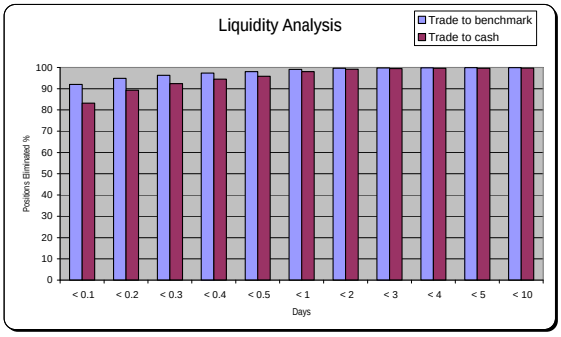
Under-weights in Large Value and Large Growth contri-buting most to stock selection risk

Japanese IT stocks are at the top of the stock-by-stock risk report

Top 10 Risk Contributors

Company Name	Mkt	Sector	CASA Style	Risk %	Port %	Bench %	Active Weight
TOKYO STEEL MANUFACTUR	JPN	Materials	Large Value	2.95	0.5	0.0	0.5
UNIDEN CORPORATION	JPN	Info Tech	Small Value	2.78	0.5	0.0	0.5
Cash Japanese Yen		Cash		2.49	1.9		1.9
INDEX CORPORATION	JPN	Info Tech	Large Growth	2.36	0.5	0.0	0.5
NOKIA OYJ	FIN	Info Tech	Large Growth	2.20		0.4	-0.4
YASKAWA ELECTRIC CORPO	JPN	Info Tech	Large Growth	2.10	0.5	0.0	0.5
KURARAY CO LTD	JPN	Materials	Large Value	1.77	0.5	0.0	0.5
DELL INC.	USA	Info Tech	Large Growth	1.72		0.4	-0.4
TAKUMA CO LTD	JPN	Industrials	Large Value	1.59	0.5	0.0	0.5
TAIYO YUDEN CO LTD	JPN	Info Tech	Large Value	1.56	0.5	0.0	0.5

The Top 10 Risk Contributors are those equity stocks contributing the largest proportions of Tracking Variance relative to the benchmark. Stock Styles, Sectors and Market of registration are also identified; and further information is also available from the full output report.



Liquidity Analysis indicates the time required to liquidate the entire portfolio or to trade back to the benchmark. It is based on a notional portfolio value of 100 million USD and based on a presumed ability to trade 20% of each share's daily volume.

No liquidity issues at this notional level of AUM

The Main Performance Reports

Now that we have seen the key features of a snapshot of the case study portfolio[1], let's focus on performance. This section introduces the main performance sheets – each will be described in detail in the dedicated sections that follow.

Overall performance analysis, comparison with audited returns, and bottom-up stock contributors may be found in the **Perf Summary** page. This includes charted and tabulated returns over a variety of time periods, both cumulatively and per month, for the portfolio and its benchmark. Top and bottom ten stock contributors are also shown over a range of user-selected periods, along with the relative returns and average active positions per stock.

The next three reports use the standard 'Brinson' decomposition to separate the performance impact of asset allocation from stock selection decisions. Contributions from currency movements and cash held are also shown on these pages.

> Country allocation decisions may be investigated in **PerfMarket.** For various user-selected periods the aggregate or per country contributions from over or underweighting countries are shown cumulatively and per month. In addition, the impact of stock selection *within* countries may also be viewed for various periods. This report is only available for multi-market analyses.
>
> Sector allocation decisions are reported in **PerfSector.** This page is analogous to the country allocation page, the sector classification being either FTSE or MSCI, depending on the user subscription/preference.
>
> **PerfStyle** separates allocations to various categories of stocks in terms of Style-based criteria. The default view, into Large Value, Small Value, Large Growth and Small Growth segments permits the separation of performance due to thematic exposures from the impact of stock selection within those groups. User-defined Styles may also be specified.

The final 'formatted' performance sheet, **SR Perf Chart**, uses the Style Research hierarchical approach to show contributions from all sources of return in one graph. The default attribution breaks down the portfolio's active return into currency, country, sector, Style and stock components, though these elements may be re-ordered or even suppressed. Furthermore, a pivot table gives the user the ability to examine the various contributions for specific dates from a variety of perspectives.

Much more information, including all the stock by stock data, may be found in the data sheets, located towards the back of the output spreadsheet, and described in Appendix P1.

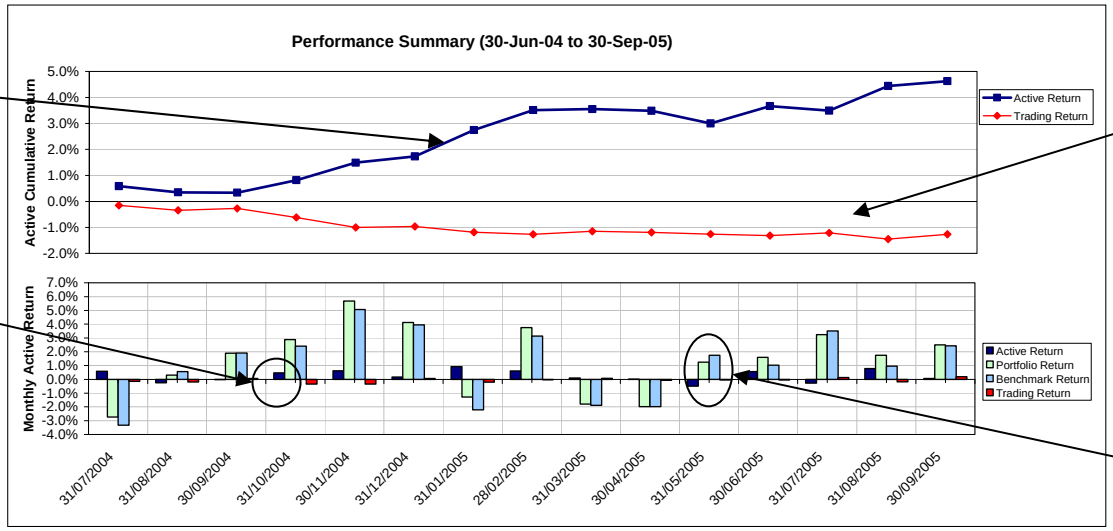
Now let's start our analysis of the case study, and look at the first of these reports, the **Perf Summary** sheet.

[1] A 'Snail Trail' analysis reveals that this is representative of the portfolio characteristics over the entire period.

Performance Overview - 'Perf Summary'

Cumulative active return in base currency

Each month, active return equals portfolio return less benchmark return



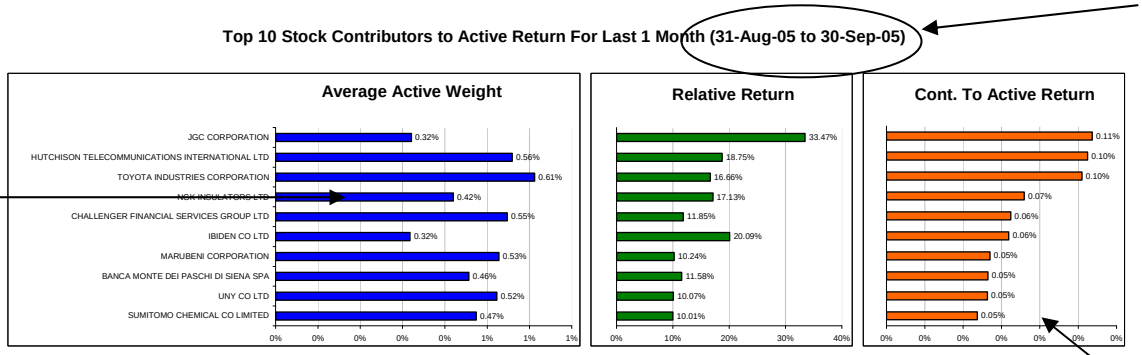
Trading return is the difference between calculated and actual portfolio returns (if loaded)

Absolute monthly base currency returns for portfolio and benchmark

Returns are tabulated over various horizons

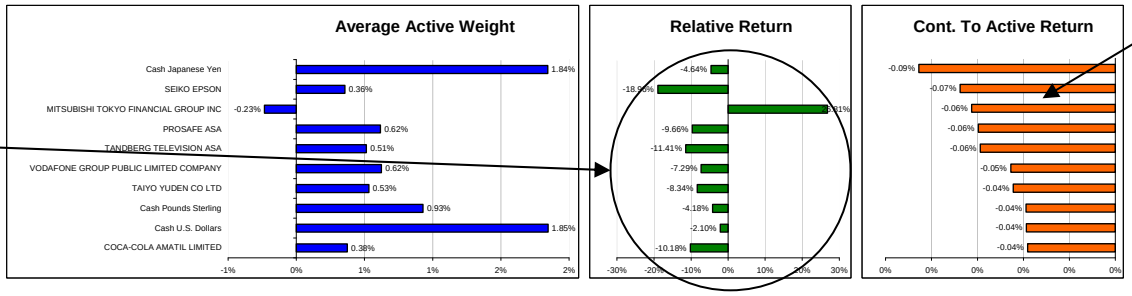
	Returns			Actual Returns	Trading Returns
	Portfolio	Benchmark	Active		
1 month	2.5%	2.4%	0.1%	2.7%	0.2%
3 months	7.7%	7.0%	0.6%	7.8%	0.1%
6 months	8.6%	7.8%	0.7%	8.5%	0.0%
12 months	23.6%	19.4%	4.2%	22.7%	-0.9%
Since Inception	22.9%	18.3%	4.6%	21.6%	-1.3%
Since Inception (Annualized)	17.9%	14.4%	3.6%	17.0%	-1.0%

Period may be selected by user



Period average of portfolio weight less benchmark weight per stock

Bottom-up stock contributions to active return over period. Ten best and worst contributors displayed



Difference between stock return and benchmark return over period

Performance Overview – 'Perf Summary'

The *Perf Summary* sheet starts with the **Performance Summary** chart, a view of the overall portfolio performance, in base currency terms. The active return (*i.e.* portfolio less benchmark return) is shown both cumulatively and monthly. Both portfolio and benchmark returns are based on monthly buy and hold portfolios. The impact of intra-month trading and transaction costs is shown in the *trading return*, which is the difference between the actual portfolio returns (if available, and loaded) and the calculated portfolio returns.

In our example, the portfolio has outperformed the benchmark, consistently adding value. Over the entire period, the portfolio returned 22.9% in US$ terms compared with 18.3% for the benchmark, an active return of +4.6% (or 3.6% *p.a.*). The best month for the portfolio was January 2005 (outperformance of +0.9%) and the worst was May 2005 (underperformance of 0.5%).

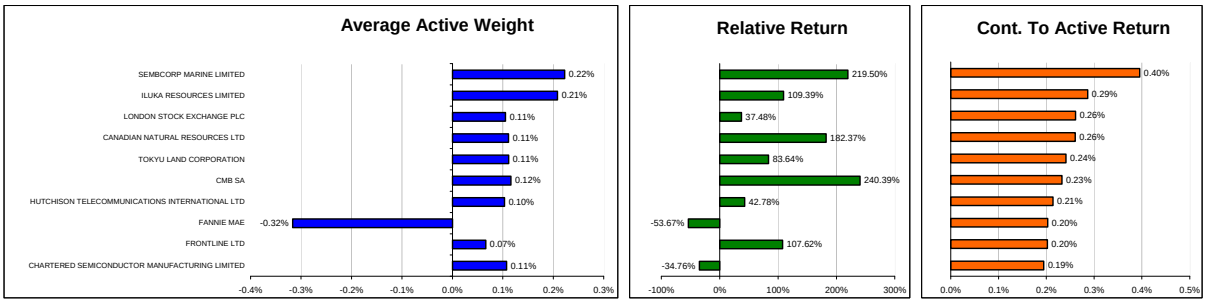
The trading return has been a consistent but relatively small drag on performance. Overall, the actual returns were some 1.3% points below the calculated returns, (1% *p.a.)* and so the outperformance has been overstated by a similar amount. However, this still means that the portfolio outperformed its benchmark by 3.3% over the period. Examination of the monthly trading returns shows that this accrued fairly evenly over the period with no particular month standing out. Therefore we can have confidence that the attribution results that follow are realistic and meaningful.

The active return is broken down into contributions per stock over the selected period. Across all stocks in the portfolio and benchmark these contributions sum to give the active return for the period. The ten best and worst contributors are displayed. In our example, the single best contributor over the latest month (Sep 2005) was *JGC Corporation* where the manager was 0.32% overweight. This stock outperformed the benchmark by 33.5%, giving rise to an 11 basis points[2] contribution to active return over the month. The portfolio's worst stock performer was *Seiko Epsom*, from an overweight position of 0.36%. This stock underperformed the benchmark by 19% over the month, leading to a -7 basis points contribution. Note also that the position in Yen cash is shown explicitly, and was the worst overall position for this period.

The user may view these 'bottom-up' stock contributions over different horizons by selecting the appropriate period from the dropdown in the output sheet. The following shows the top ten contributors over the entire period for our case study.

[2] Remember that 1 basis point = 0.01%, or a hundredth of 1%.

Top 10 Stock Contributors to Active Return For Entire Period (30-Jun-04 to 30-Sep-05)



In this case the best single stock contribution to return was from *Sembcorp Marine*, which beat the benchmark by 220% over the period, and contributed 40 basis points to the overall active return. The average active weight was 0.22%.

Attribution by Market – 'PerfMarket'

The *PerfMarket* sheet is the first of the Brinson-based performance attributions. It separates out the impact of country asset allocation decisions from the stock selection contribution within countries. A manager who is overweight a country that has outperformed the benchmark will have a positive allocation contribution, as will a manager with an underweight position in an underperforming country. Negative contributions arise when a manager is over(under)weight an under(out)performing country.

Starting with the performance over the entire period, the **Market Attribution** chart shows that the overall relative return of 4.6%[3] may be broken up into 2.2% from country allocation, 1.7% from stock selection within countries, 1.1% from interaction, 0.0% from currency, and -0.4% from cash. The chart also reveals that currency was important during the first six months, and that both country allocation and stock picking have been positive since that time.

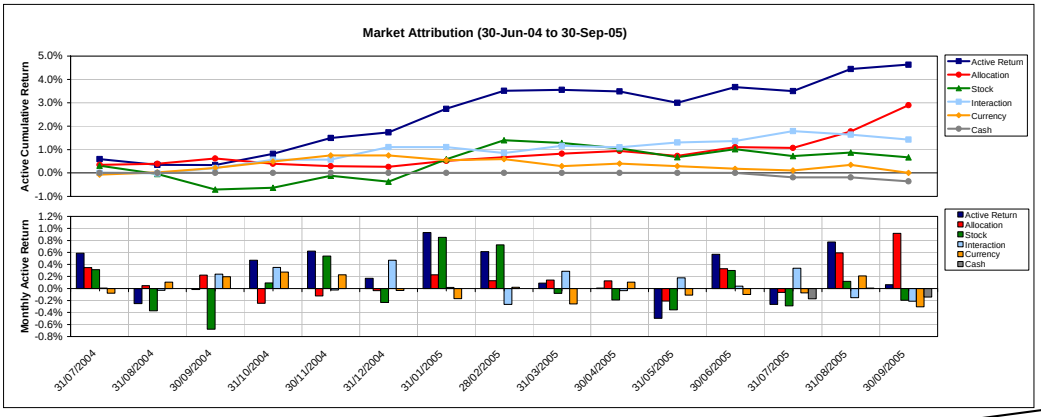
The **Contribution by Market** chart in the middle of *PerfMarket* breaks this down into contributions for each country. In our example, the overweight of 6.2% in Japan, a market that has outperformed the benchmark by 8.8% over the latest month, produces an allocation contribution of 0.5%. Similarly, the -15.9% underweight in the US, which has underperformed by 2.4%, also gives a positive contribution to country allocation (of +0.4%). Across all countries, the allocation contributions sum to 0.9% for the latest month, as displayed in the **Market Attribution** chart at the top of the sheet.

The stock selection contribution compares the portfolio return with that of the benchmark, per country, and weights this difference by the benchmark allocation to that country. For the case study, the contribution of -0.3% for stocks in Japan derives from the active return in Japan, -3.3%[4], times the benchmark weight, 9.9%[4]. Across all

[3] The numbers shown are not readable from the extract of the Excel sheet shown on the following page. Within the actual Excel output the values may be viewed by 'hovering' above the lines on the chart.
[4] See Appendix P1 for information on where to find this underlying data

Attribution by Market - 'PerfMarket'

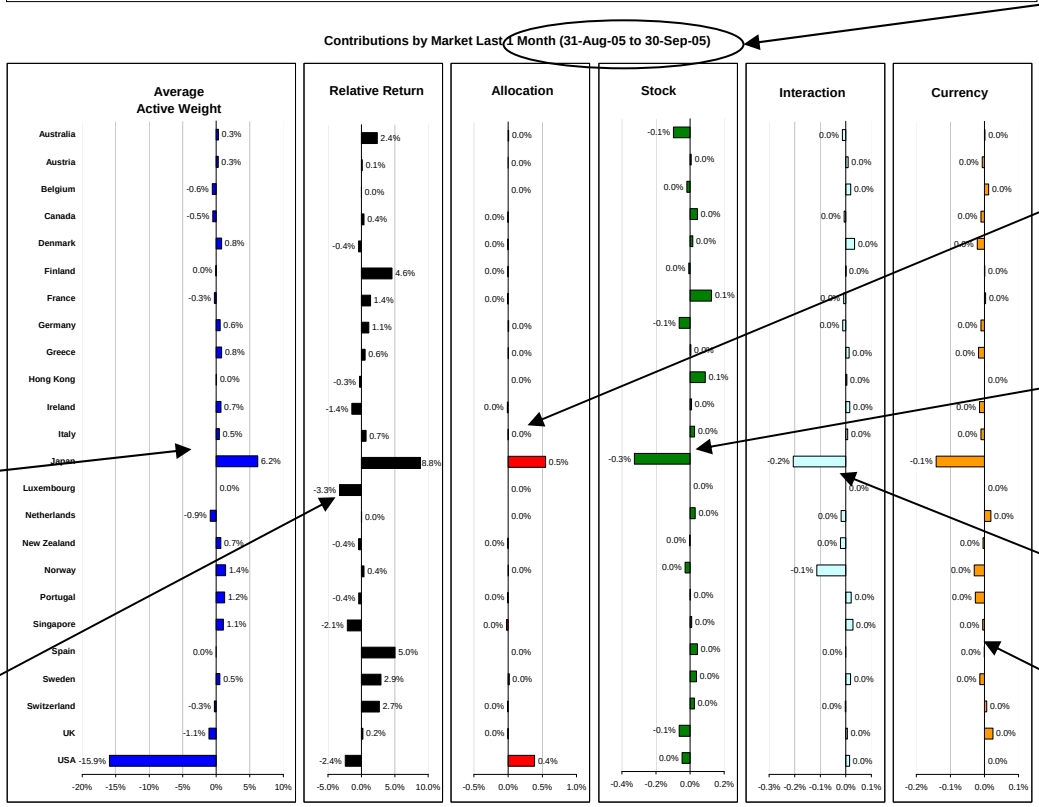
Brinson de-composition of active returns into country, stock and interaction terms



Contributions from cash holdings and currency are shown explicitly

Time horizon chosen by user

For each country a variety of contributions are shown for the chosen period



Per country contribution to asset allocation

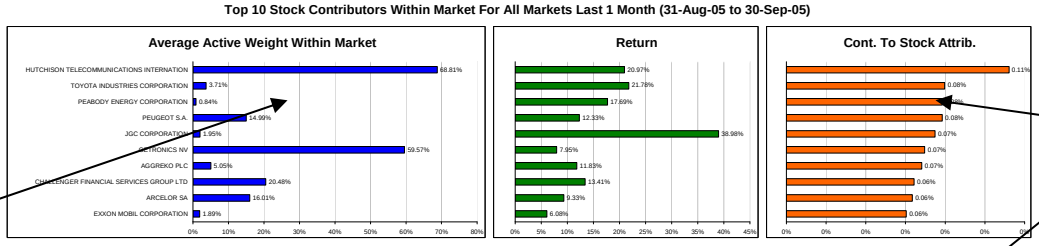
Per country contribution to stock selection within countries

Average active country bet over the period

Per country interaction between country decisions and intra-country stock selection

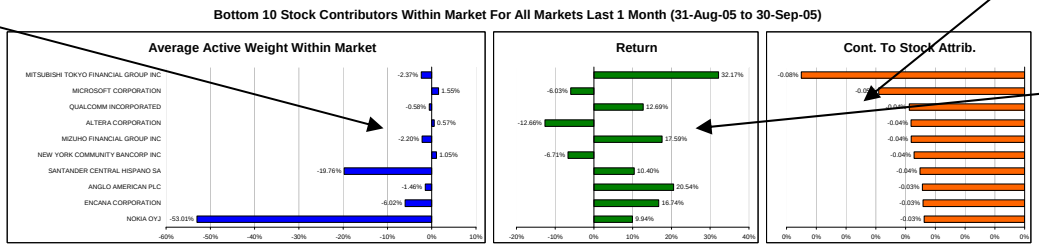
Benchmark country return relative to overall benchmark

Per country contribution from currency effects



Best/worst stock contributions within countries

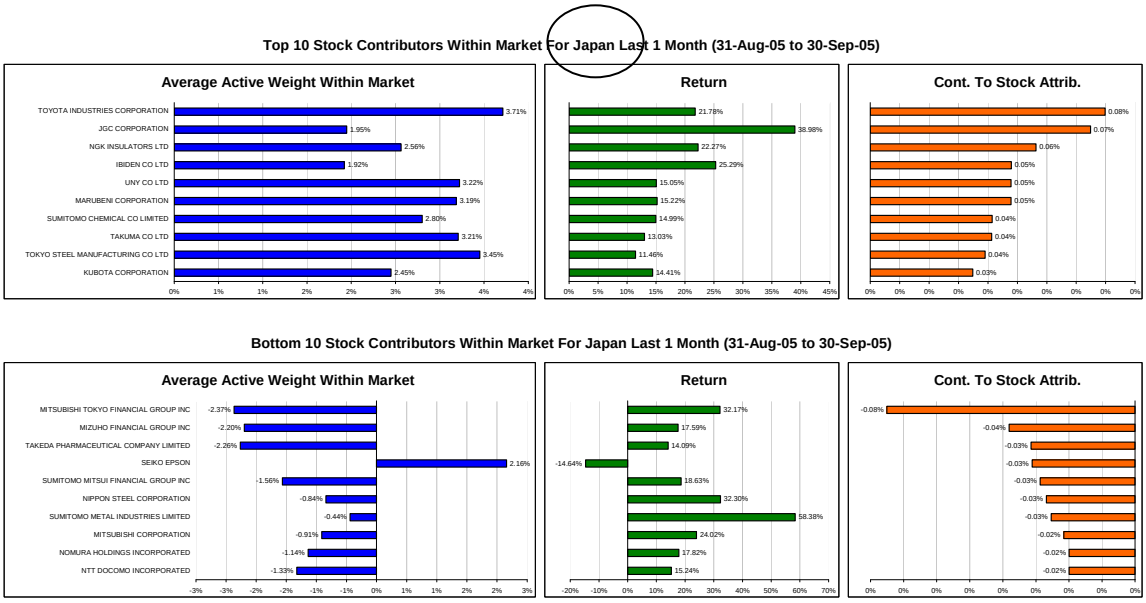
Average active positions within countries



Stock returns relative to corresponding benchmark country return

countries the stock selection contributions add to -0.2% for the latest month, as shown in the **Market Attribution** chart.

More detail at the stock level may be found in the **Top/Bottom 10 Contributors within Markets** chart at the bottom of the *PerfMarket* sheet. This may be viewed across all countries, or for specific countries. In view of the importance of stock picking in Japan for this portfolio, we select this market and show the table below:



This helps to pinpoint the areas of success, e.g. the overweight in *Toyota* was the best stock selection decision in Japan over this period.

Returning to the **Contributions by Market** chart, the next column along is the interaction contribution. This is a combination of country and stock attribution effects. It combines the active position in a country with the stock selection contribution in that country. It is positive when a manager is overweighting a country where stock selection is good, or conversely, underweighting one where stock selection is poor. Other combinations (e.g. overweighting a market where stock selection is weak) will give rise to negative interaction terms. In our example the 6.2% overweight in Japan, where the manager has underperformed by 3.3%, yields an interaction term of -0.2%. Across all countries, these terms sum to -0.2%, offsetting the stock selection contribution for the latest month.

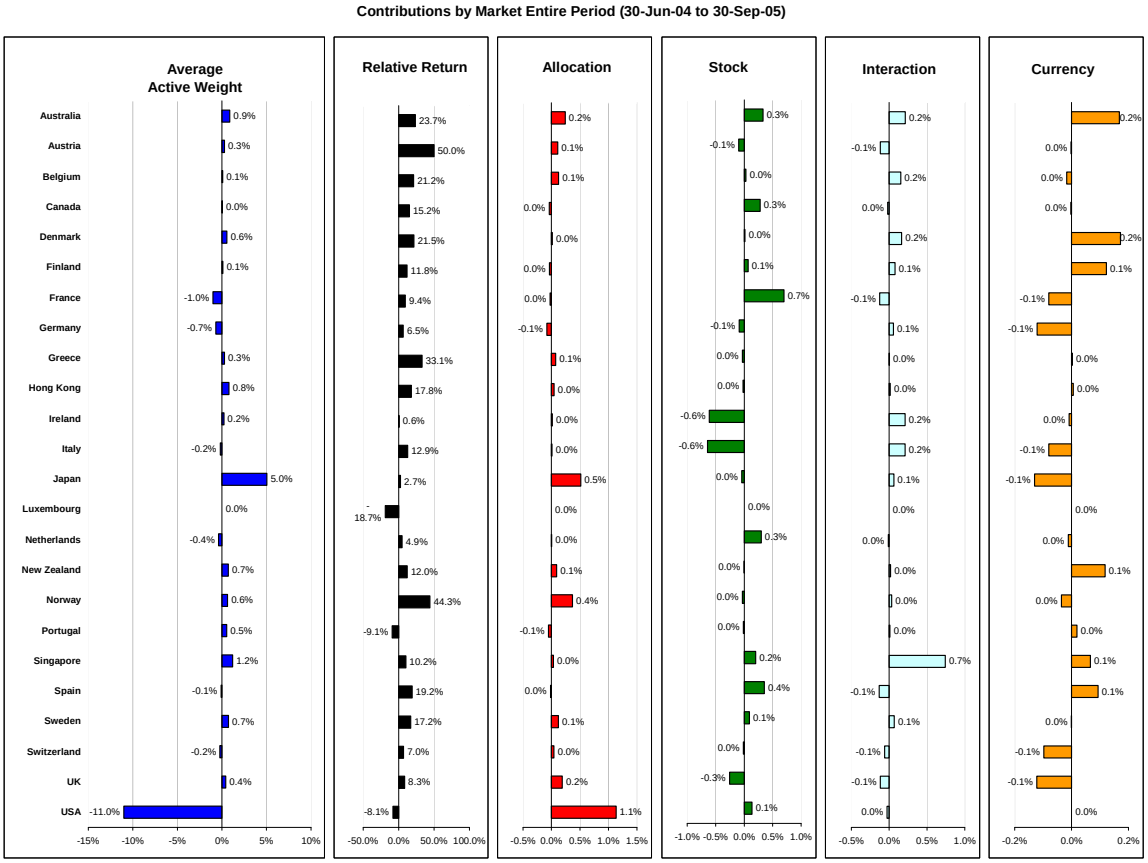
The above returns for allocation, stock selection and interaction are in local currency terms. The impact of currency is shown separately, in the next column of the chart. In our example, the overweight to Japan is a cost to the portfolio in currency terms, as the Yen fell slightly against the dollar over the latest month, giving a -2.2% return for a $ based investor on Japanese investments. The impact on the active return is approximately the overweight in Japan times this currency return, i.e. -2.2% * 6.2% = 0.14% as shown[5].

[5] See equation A22 in Appendix P2

Finally, the cash return corresponds to the performance impact of holding cash in the portfolio. For the latest month, the portfolio cash weight was 4.6% vs. zero in the benchmark. The return of this, relative to the benchmark, is -3.1%, and this gives rise to a -0.14% contribution.

Note that the five components, allocation, stock selection, interaction, currency and cash add up to the overall base currency active return each month.

In the output sheet it is possible to examine the above components over different periods. The following shows the **Contributions by Market** chart for the entire period.



This view helps to summarize where the performance has come from over the entire period. From the **Market Attribution** chart it is clear that country allocation was a positive contribution – here we see that the underweight in the US is the main reason. Amongst other positions, the overweights in Japan and Norway (which performed particularly well) also helped.

Stock selection was good in a number of countries, but France, Australia and Spain were among the better areas. In contrast, Ireland and Italy had less successful stock selection. The overall interaction term is dominated by Singapore, where an overweight position reinforced the impact of good stock selection.

Much more data behind the charts in *PerfMarket* may be found in the data sheets towards the back of the Excel output sheet, especially in *Perf Attrib Data* and *Agg Perf History*. See Appendix P1 for details.

Attribution by Sector – 'PerfSector'

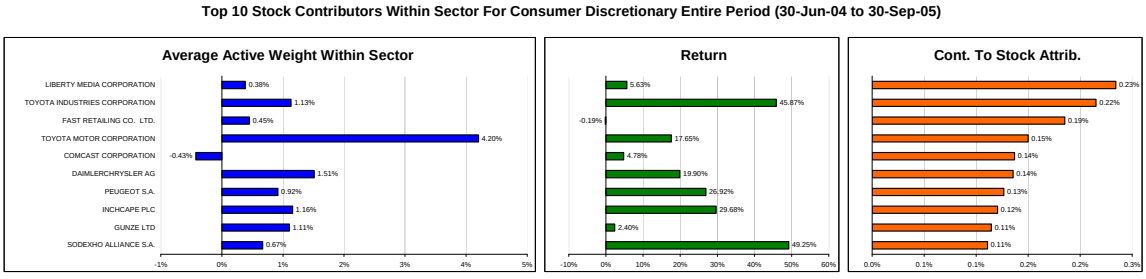
The second Brinson performance attribution sheet focuses on sector allocation and stock picking within sectors. The performance breakdown is entirely analogous to that used in the previous section, except stocks are now distinguished by their sector rather than country membership.

Therefore, the components of active return are now: sector allocation, stock picking within sectors, interaction, currency and cash. The latter two components are identical to those in the country allocation sheet.

Recall that for our case study the sector bets are, by and large, quite moderate. This has translated into rather small contributions from sector allocation. In the **Sector Attribution** chart at the top of the following page the red line representing the return due to sector bets has not deviated far from zero contribution. But the green, stock picking within sectors, line, has virtually mirrored the overall active return. Therefore, for this portfolio at least, the sector attribution has revealed that sector allocation was not an important determinant of performance.

That said, this view *does* permit investigation of stock picking results sector by sector. As can be seen from the chart **Contributions by Sector**, over the latest month the best stock picking results are found in the Consumer Discretionary and Industrials sectors. Stock picking was less successful in Financials.

As in the *PerfMarket* sheet, the time horizon may be changed, and the **Top/Bottom 10 Stock Contributors** chart may be edited to highlight one sector at a time. As an example, here is a view of the best picks within the *Consumer Discretionary* sector over the whole period.



In this way, the overall performance may be broken down into areas of interest, and then these periods or sectors can be further analyzed right down to the stock level. As usual, all the background data, and more, may be found in the data sheets – see Appendix P1 for details.

Attribution by Sector – 'PerfSector'

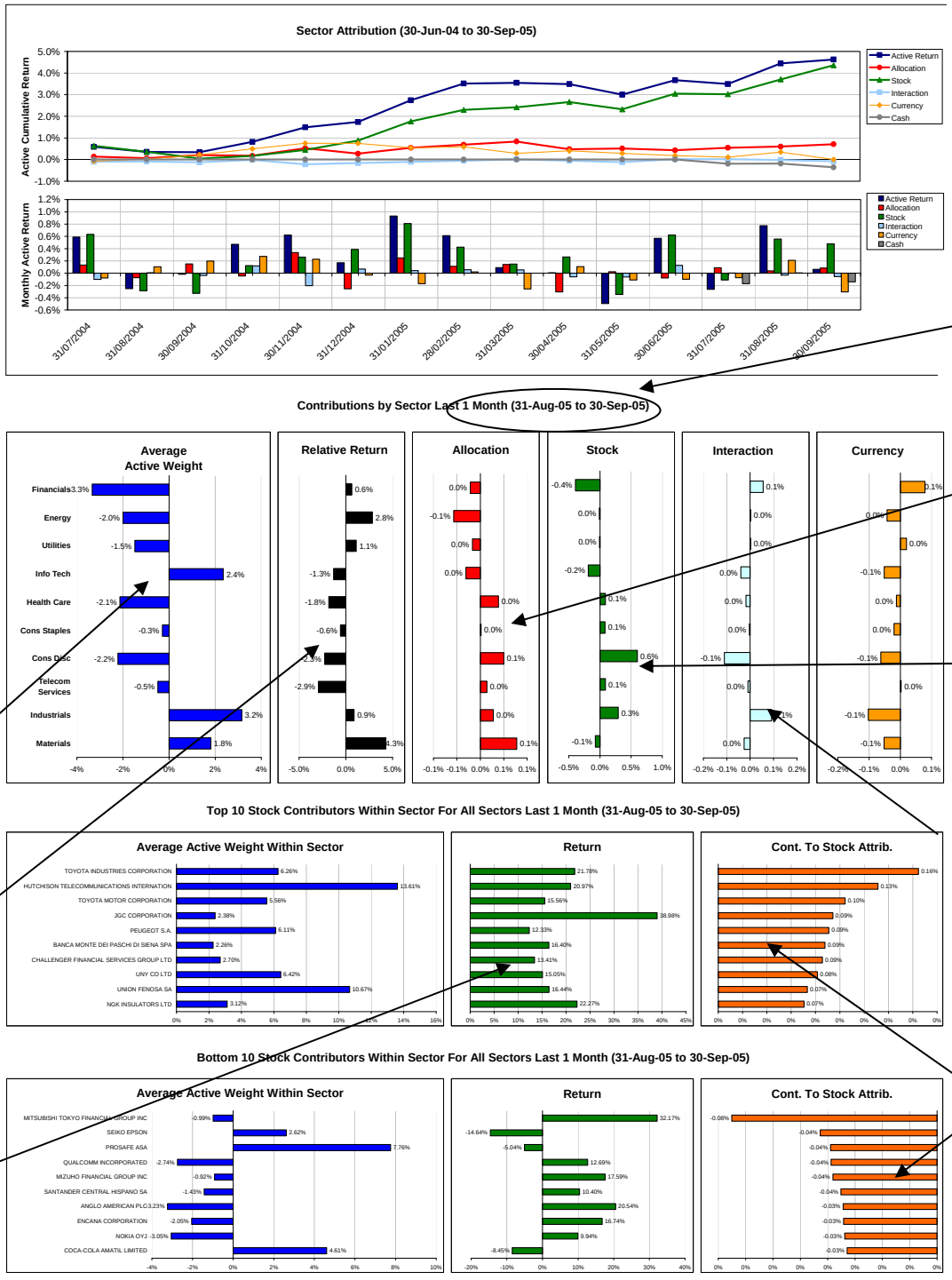
Brinson de-composition of active returns into sector, stock and interaction terms

For each sector a variety of contributions are shown for the chosen period

Average active sector bet over the period

Benchmark sector return relative to overall benchmark

Stock returns relative to corresponding benchmark sector return



Contributions from cash holdings and currency are shown explicitly

Time horizon chosen by user

Per sector contribution to asset allocation

Per sector contribution to stock selection within sectors

Per sector interaction between sector decisions and intra-sector stock selection

Best/worst stock contributions within sectors

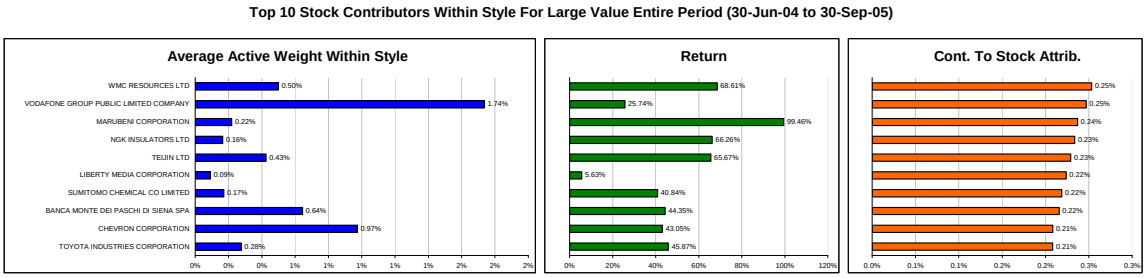
Attribution by Style – 'PerfStyle'

The final Brinson-based attribution uses Styles to carve up the analysis region. Allocation to Style groupings is then separated from stock picking within Styles.

The default settings split all the stocks in the analysis region first by market cap (top 80% = 'Large', bottom 20% = 'Small'), and then these groups are further divided by book to price (top 50% = 'Value', bottom 50% = 'Growth'). Hence four segments are created, such as the 40% of the market in 'Large Growth' stocks. Note that these groupings may be changed by the user – see later.

Given these Style-based groups, the impact of allocation to them vs. stock selection within them, may be assessed. As in the cases of country and sector decompositions, there are five terms: allocation, selection, interaction, currency and cash. The **Style Attribution** chart on the following page displays this breakdown for the case study. The red line shows that for most of the period Style allocation has been positive. It is clear from the **Contribution by Style** chart (set to show the entire period), that this portfolio was on average overweight small caps, and especially underweight Large Growth. This has given rise to positive contributions from all Styles except Large Value, where the underweight has hurt performance a little.

Stock picking has been strong within all Styles with the best returns found at the large cap end. The following view shows the best contributors within Large Value over the period:



For example, *WHC Resources*, which has been 0.5% overweight on average within Large Value, and which outperformed that category by 69% over the period, contributes around 25basis points towards the stock selection component.

Attribution by Style – 'PerfStyle'

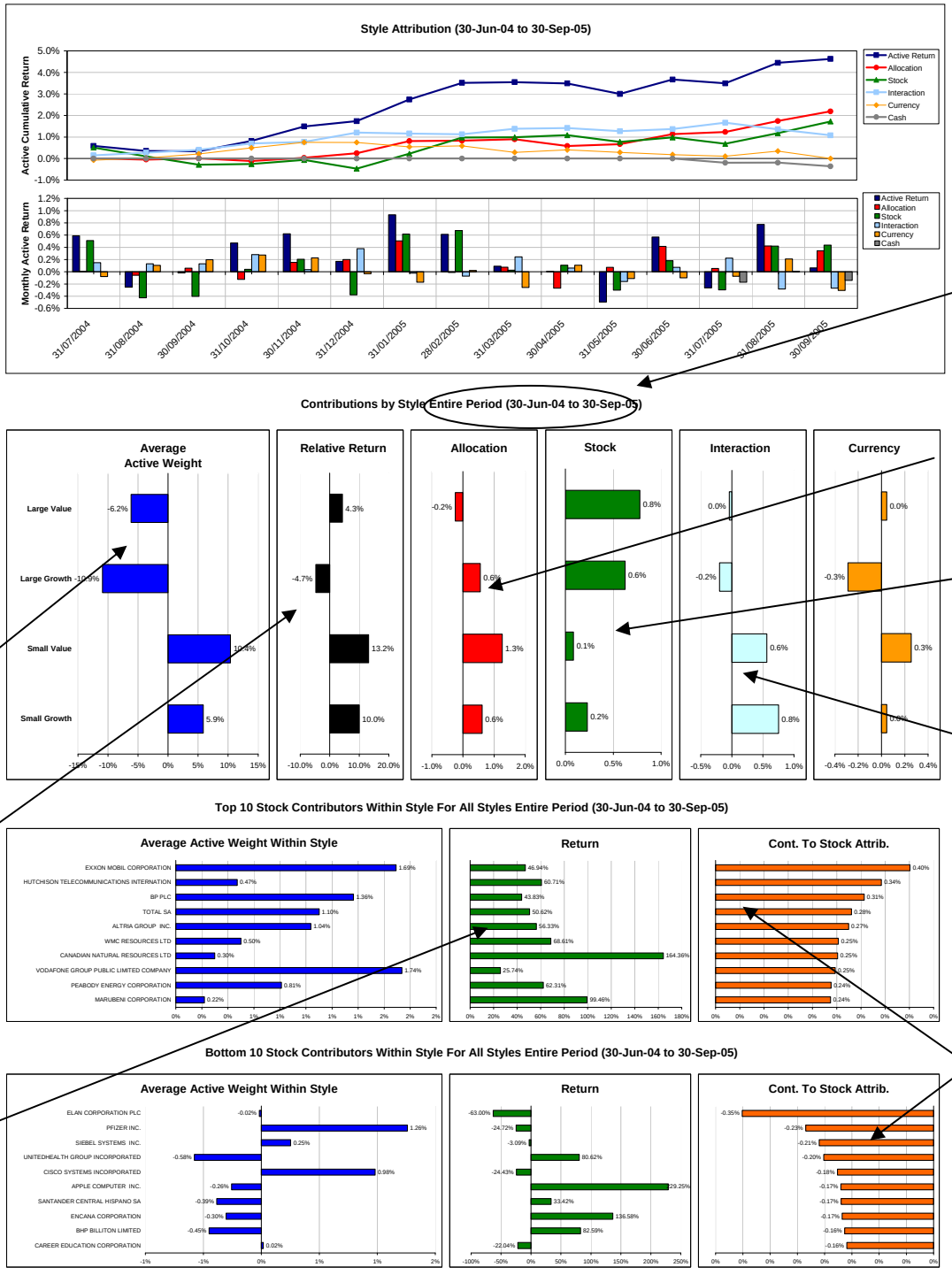
Brinson de-composition of active returns into Style, stock and interaction terms

For each Style a variety of contributions are shown for the chosen period

Average active Style bet over the period

Benchmark Style return relative to overall benchmark

Stock returns relative to corresponding benchmark Style return



Contributions from cash holdings and currency are shown explicitly

Time horizon chosen by user

Per Style contribution to asset allocation

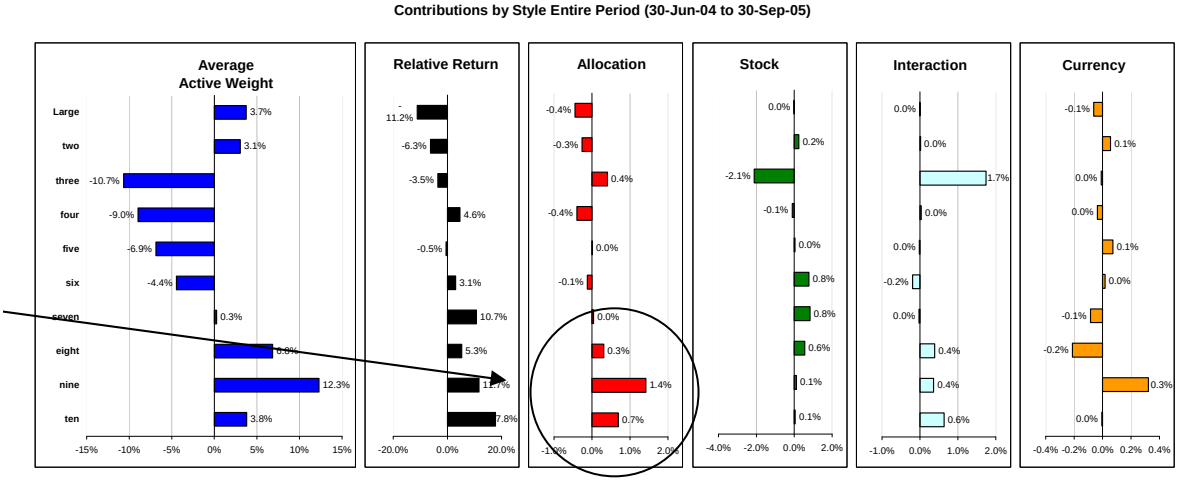
Per Style contribution to stock selection within Styles

Per Style interaction between Style decisions and intra-Style stock selection

Best/worst stock contributions within Styles

Styles may be defined flexibly by the user. This is a very powerful feature. We could, for example, examine the performance with respect to different criteria. First let's look at using just one factor to partition the market. In the following example, the market has been divided into deciles of market cap and the analysis has been re-run to investigate whether size was important irrespective of any Value/Growth influences.

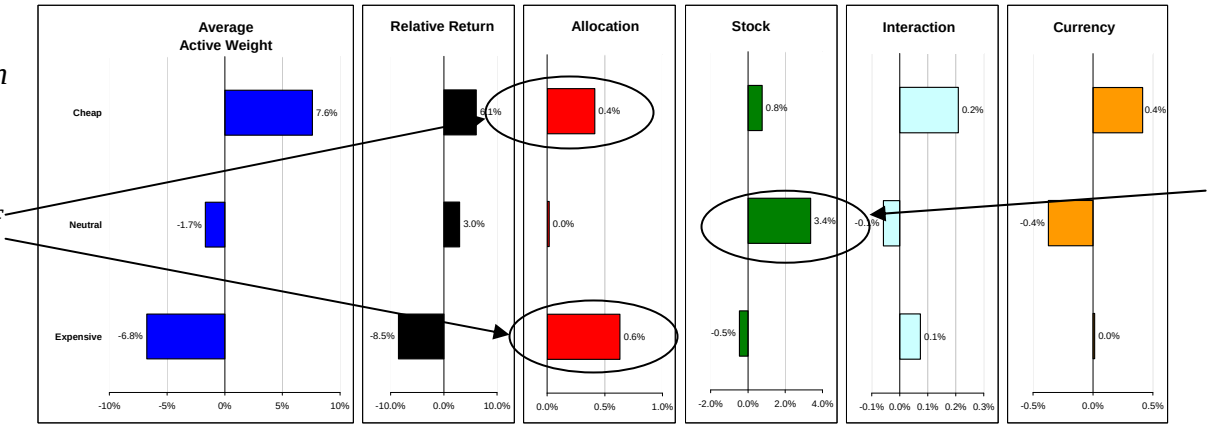
The overweight to the smaller caps has paid off



The above graph shows that the main allocation benefits have been through overweighting the bottom three deciles of market cap, although stock selection has also been reasonable in most of these deciles. The manager has been slightly overweight at the largest end of the market cap spectrum, and this has hurt performance.

We could also define Style in yet another way. The 'Front Page' summary revealed significant tilts towards higher Book-to-Price. The following analysis splits the market up into three segments using a 40/20/40 cap-weighted breakdown by Book-to-Price.

Both the overweight in Value and underweight in Growth have paid off



Stock picking best in 'Neutral'

This confirms that the allocation to cheaper stocks (and away from 'expensive') has paid off over this period. It also reveals that the best stock selection was within stocks with a 'Neutral' Book-to-Price valuation relative to the market.

Other cuts are available of course, incorporating any of the 30 plus Style factors (as part of either a one or two factor split).

For all these Brinson type cases, it is possible to drill down to the stock level for each month, and much of the underlying data is available in sheet '*Agg Perf History*' – see Appendix P1 for details. Note especially the discussion on pages 25-26 on how to access the underlying data for the Brinson components through time e.g. to view the impact of stock selection within Large Growth over the period, and so on.

Attribution by Hierarchy - 'SR Perf Chart'

The hierarchical method of performance attribution is based on a simple categorization of returns. When the default settings are used, this breaks returns into the following five components: currency, country, sector, style and stock specific (equity). For example, Microsoft's return in Euro terms is decomposed into Euro/$, US market, US IT, Large Growth US IT and Microsoft stock specific returns.[6]

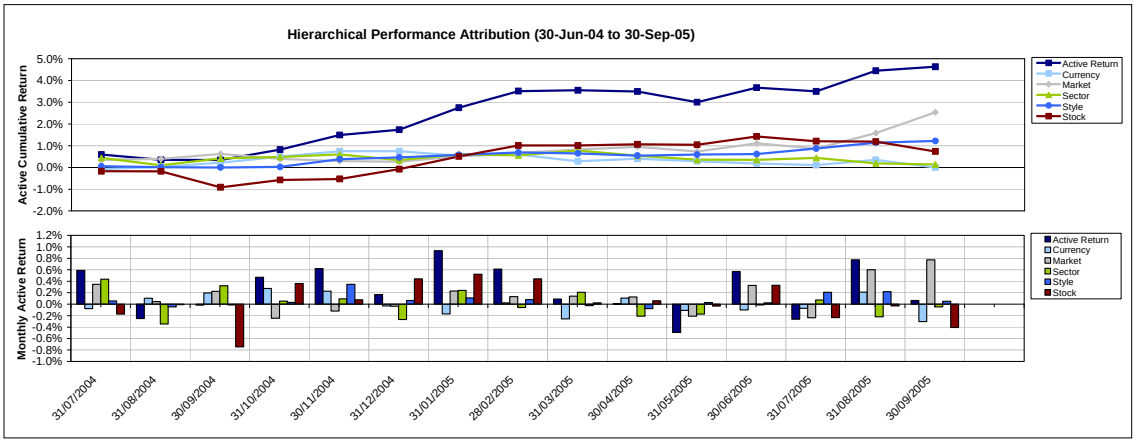
There are a number of benefits in using this approach. Firstly, it allows the investigation of all the contributing performance drivers at the same time – unlike the Brinson approach which examines one element at a time. Secondly, the returns are nested, so that, for example, the effect of sectors within countries, or Styles within sectors, can be examined. Furthermore, the order of the nesting can be varied, or layers removed. For example, Styles can be nested above sectors; and the market level can be left out completely.

Another aspect is that the default breakdown mirrors very closely the SRPA risk decomposition. This is useful in that it provides an ideal way of seeing whether risks taken have paid off, and as such, can give insights into the 'efficiency' of the investment process.

In addition to the ordering flexibility, the Style definitions can also be selected by the user. The default style breakdown is into Large Value, Large Growth, Small Value and Small Growth, as detailed in the Brinson section. This leads to a powerful method of separating out the important elements of return.

Here's the **Hierarchical Performance Attribution** chart (taken from the top of *SR Perf Chart*), for the case study:

[6] See the discussion in Appendix P2 for the detailed formulae employed.



In this example the default settings for ordering the components of return and the definition of Style have been used. It is clear that the approach captures much of what we've seen in the individual Brinson analyses: a good contribution from country allocation; nothing much from sector allocations within countries; consistent value added from Style (within sector-and-country); and positive stock selection, especially over the last year. It is worth noting that this view will not *necessarily* echo the findings in the Brinson analyses. For example, in the Brinson sector analysis the sectors will be pan region (in our case global) sectors, as opposed to local sectors (i.e. within country) in the hierarchical analysis. Likewise, the Brinson Style analysis employs global Styles, whereas in this default hierarchical analysis the Styles are defined within sectors within countries.

Also included in this report is a 'pivot table', which allows interrogation of the sources of performance across the underlying components. For example, in the above chart, it is clear that country allocation contributed strongly over the latest month. The default pivot table, below, reveals that 0.78% was added at the country level:

Detailed Performance Attribution

Time Frame	Last 1 Month (31-Aug-05 to 30-Sep-05)
Market	(All)
Sector	(All)
Style	(All)

Return Attribution	
Level	Total
Currency	-0.30%
Market	0.78%
Sector	-0.05%
Style	0.05%
Stock	-0.41%
Grand Total	0.06%

Country allocation successful

However, by simple manipulation (in this case by 'dragging' the *Markets* label from the top table to the numerical section of the pivot table), the sources of that country return may be viewed. Here's an extract of that view[7]:

[7] A number of markets have been left out of this table for display purposes.

Return Attribution	Market											
Level	AUS	AUT	BEL	CAN	Cash	CHE	IRL	ITA	JPN	SWE	USA	Grand Total
Currency	0.00%	-0.01%	0.01%	-0.01%	-0.06%	0.01%	-0.01%	-0.01%	-0.14%	-0.01%		-0.30%
Market	0.01%	0.00%	0.00%	0.00%	-0.14%	-0.01%	-0.01%	0.00%	0.55%	0.02%	0.39%	0.78%
Sector	-0.02%	0.02%	0.00%	-0.02%		0.01%	-0.02%	0.01%	-0.26%	0.02%	0.11%	-0.05%
Style	-0.02%	0.01%	0.00%	0.00%		0.00%	0.02%	0.01%	-0.08%	0.00%	0.01%	0.05%
Stock	-0.07%	-0.02%	0.00%	0.05%		0.02%	0.02%	0.02%	-0.19%	0.03%	-0.15%	-0.41%
Grand Total	-0.10%	0.01%	0.01%	0.02%	-0.20%	0.02%	0.00%	0.03%	-0.13%	0.06%	0.35%	0.06%

Note that the positions in the US and Japan are the main contributors to this return. It is also possible within this table to see the per country contributions to the other elements too. For example, most of the negative return from currency comes from Japan (-0.14%, as we saw in the discussion on the Brinson methodology).

Other time periods may be viewed. From the main **Hierarchical Performance Attribution** chart it appears that stock selection has been weak over the past quarter. The following table focuses on this period, and views the contributions by Style:

Return Attribution	Style					
Level	Large Growth	Large Value	No Style	Small Growth	Small Value	Grand Total
Currency	-0.04%	-0.04%	-0.05%	-0.03%	-0.02%	-0.17%
Market	0.52%	0.71%	-0.32%	0.04%	0.25%	1.20%
Sector	-0.15%	-0.01%	0.00%	-0.02%	0.02%	-0.21%
Style	0.10%	0.02%	0.01%	0.46%	-0.10%	0.50%
Stock	-0.14%	-0.69%	0.00%	-0.13%	0.25%	-0.70%
Grand Total	0.30%	-0.01%	-0.36%	0.33%	0.36%	0.61%

Stock selection was weak in Large Value last quarter

The 70 basis points lost through stock selection in this period came mainly from stock selection in Large Value. Further detail on the contributions per *stock* may be found in the data tables in the '*Perf Stock Level Data*' sheet of the output.

In the context of the case study, we have seen that both country allocation (chiefly underweight the US), and Style (overweight small caps) have been beneficial. Perhaps only one of these performance drivers was operating. For example, maybe the small cap returns were just a consequence of being underweight a large market such as the US. Or perhaps the converse is the case – the benefit of being underweight the US in this period was not a country effect, rather a manifestation of the small cap effect. We can investigate this by switching off the sector level in our analysis and examining performance in terms of countries, and Styles (e.g. size quintiles) within countries, and separately, Styles first, then countries within Styles. Here's the output table for the 'Market then Style' case:

Return Attribution	Style						
Level	a Large	b	c	d	e Small	No Style	Grand Total
Currency	0.04%	0.12%	-0.03%	-0.09%	0.02%	-0.06%	0.00%
Market	-0.64%	0.16%	0.76%	1.63%	0.99%	-0.36%	2.53%
Style	-0.06%	0.52%	-0.45%	0.26%	0.59%		0.87%
Stock	-0.41%	-0.67%	0.43%	1.22%	0.62%		1.20%
Grand Total	-1.07%	0.14%	0.71%	3.02%	2.22%	-0.42%	4.60%

Styles are still important within countries

The last column shows that the 4.60% entire period active return has positive contributions from Style even after countries have been taken into account. Much of the Style allocation return came from the small cap end.

This table has the order reversed – Styles, then markets:

Note the revised order

Return Attribution	Style						
Level	a Large	b	c	d	e Small	No Style	Grand Total
Currency	-0.01%	-0.05%	0.09%	-0.30%	0.32%	-0.06%	0.00%
Style	-0.57%	-0.09%	-0.14%	0.53%	1.97%		1.70%
Market	0.21%	0.32%	-0.04%	0.36%	1.01%	-0.36%	1.49%
Stock	-0.11%	-0.67%	0.67%	1.22%	0.30%		1.41%
Grand Total	-0.48%	-0.48%	0.58%	1.81%	3.59%	-0.42%	4.60%

Countries are still important after Styles

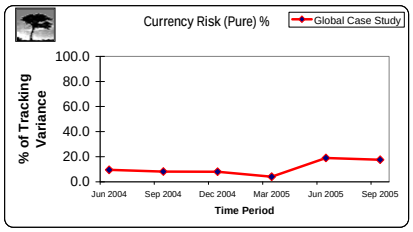
From this view, Style accounts for more of the overall return, but countries continue to add to performance. The conclusion is that there are at least two drivers of performance, with both countries and Styles contributing, irrespective of the order of the nesting.

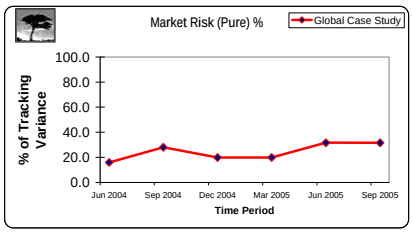
It is worth noting that the hierarchical and Brinson approaches are equivalent if only one layer of nesting is selected. For example, if the hierarchical method is used with just the market layer (i.e. sectors and styles 'switched off'), the contributions from country allocation will be identical in the two cases, and the hierarchical stock selection contribution is the same as the Brinson stock selection and interaction combined. See the discussion on page 46 for details.

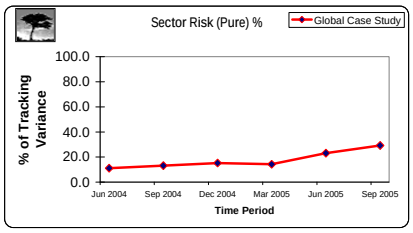
Risk Contribution vs. Hierarchical Performance

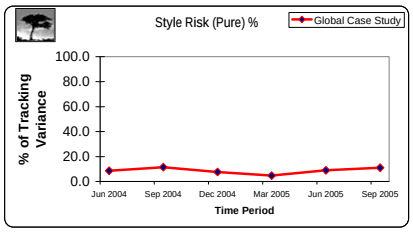
Given that the default hierarchy used in the Performance Attribution Module mirrors the same breakdown employed in the risk report in the main SRPA output, the sources of risk and return may be compared.

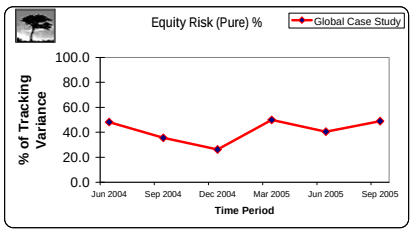
Here's the evolution of the active risk, taken from the 'Risk Attribute' sheet from a 'Snail Trail' analysis for the case study portfolio.

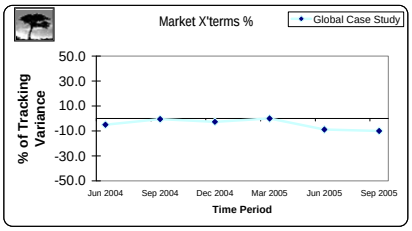


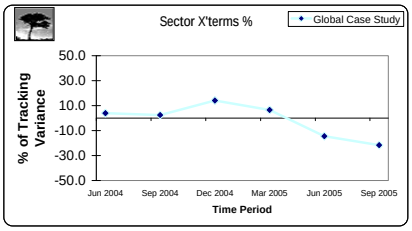


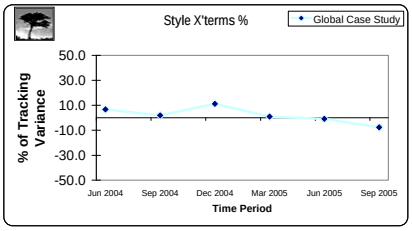


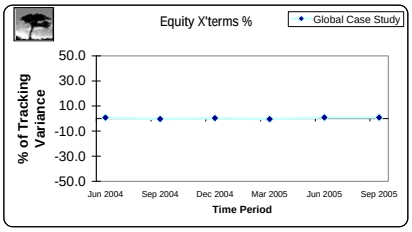












On average, over one third of the risk has come from stock selection. Closer examination (not shown), reveals that half of this comes from Large Growth and 40% from Large Value. The question is: Has this risk exposure paid off?

The answer may be found in the hierarchical pivot table for the entire period, broken out by Style:

Return Attribution	Style					
Level	Large Growth	Large Value	No Style	Small Growth	Small Value	Grand Total
Currency	0.15%	-0.10%	-0.06%	-0.04%	0.04%	0.00%
Market	0.79%	1.42%	-0.35%	0.22%	0.46%	2.53%
Sector	-0.42%	0.48%	0.00%	-0.04%	0.13%	0.14%
Style	-0.44%	-0.03%	0.00%	1.10%	-0.28%	1.22%
Stock	-0.36%	0.05%	0.00%	0.37%	0.67%	0.73%
Grand Total	0.58%	1.82%	-0.41%	1.61%	1.03%	4.63%

Poor stock selection within Large Growth revealed

Good returns overall, and from each component

Large Growth was the *only* area where stock selection was weak for this manager, and Large Value was neutral. Likewise, as we have seen elsewhere, country allocation has been good for this portfolio (2.53% out of the 4.63% active return). Interestingly the snail trail charts indicate only around 20% of risk was taken in this area.

In this way, combining *ex ante* risk and *ex post* performance attribution can help in the examination of the 'operational efficiency' of a portfolio management process. This clearly has applications both for refining investment processes and for manager assessment.

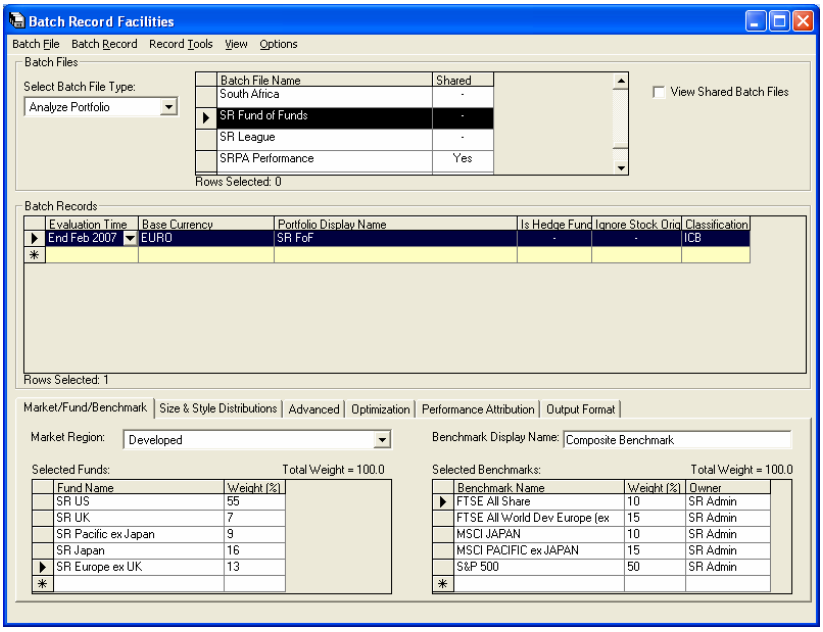
Fund of Funds Attribution – 'PerfFund'

So far we have focused on a single portfolio against a benchmark. If the analysis is for a fund of funds/multi-manager structure, then the default analysis will aggregate all the underlying holdings, and the attributions by Country, Sector, Style and Stocks will be performed for the consolidated portfolio in the usual way. However, there is also a user option to select a *fund of funds attribution* which will include extra analysis from a sub-fund perspective. This analysis separates out the performance due to the allocation of funds from that of the funds' individual performances against their respective benchmarks. The user is required to identify the **fund benchmark** associated with each fund. For example, the fund benchmark chosen for a US Large Cap Value portfolio might be the Russell 1000 Value index; or the fund benchmark for a continental European fund might be the FTSE AW Europe ex UK index. In addition, the overall fund of funds benchmark needs to be specified.

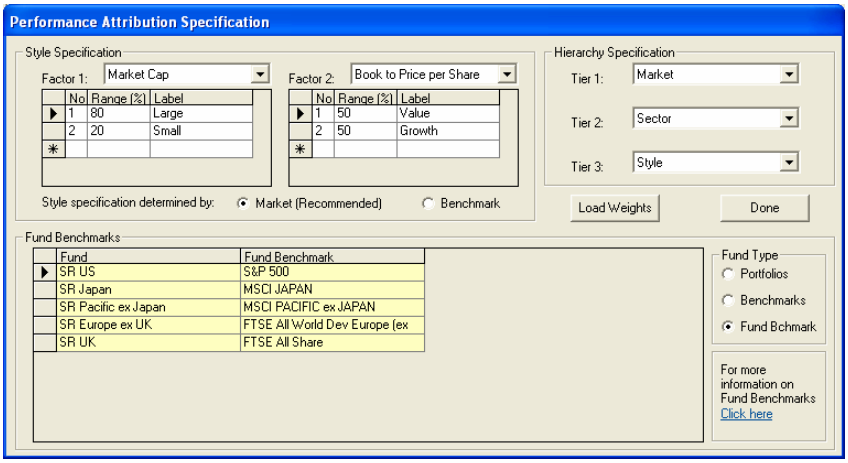
There are two variations of this analysis. The first is the special case where the overall benchmark is a composite of the fund benchmarks. Using the above example, an appropriate overall benchmark is: 60% Russell 1000 Value/40% FTSE AW Europe ex UK. An alternative might be 100% FTSE AW Europe ex UK. However, 40% Russell 1000 Value/20% S&P 500/40% FTSE AW Europe ex UK would not be appropriate as it contains a benchmark (S&P500) that is not one of the fund benchmarks. In this case, the second variation is used. The differences in the two analyses will be clarified below.

1. Benchmark constructed from fund benchmarks.

We start with the special case where the fund benchmarks entirely constitute the overall benchmark. The screenshot below shows such an example:



Although this example is based on regional funds and benchmarks, Country, Sector, Style or some combination of these are also possibilities. At this stage, the sub-funds have not been linked to their appropriate fund benchmarks. That needs to be done from the Detailed Settings section of the Performance Attribution tab (this can also be accessed by users who do not have a license for the Performance Attribution module):



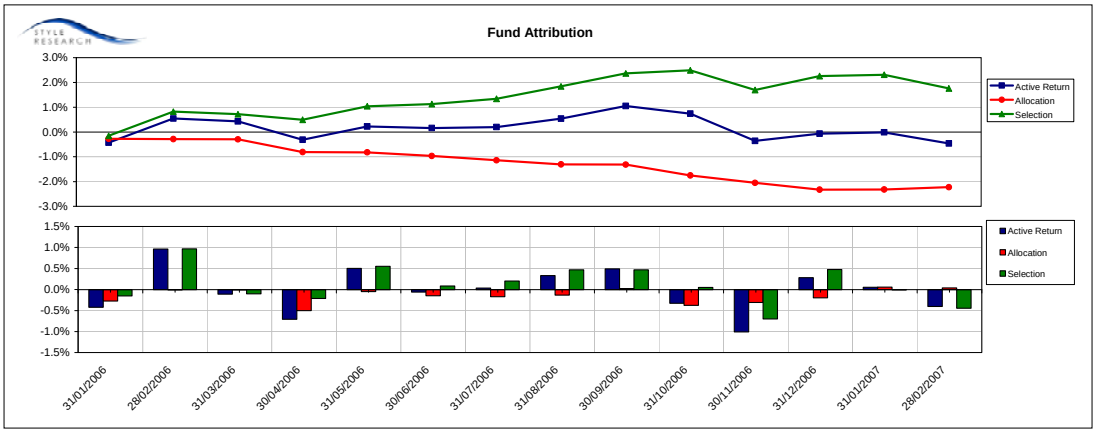
Once this has been set up, there is a clear association of each fund with its fund benchmark. Note that more than one fund may be associated with a particular fund benchmark (but not vice versa); see later. The weights of funds and benchmarks may also vary through time, as set by the user. Notice that the overall benchmark is a mix of these fund benchmarks. In this case, all the fund benchmarks are present in the overall benchmark mix, but this is not essential.

The overall performance is tabulated in the Perf Summary sheet:

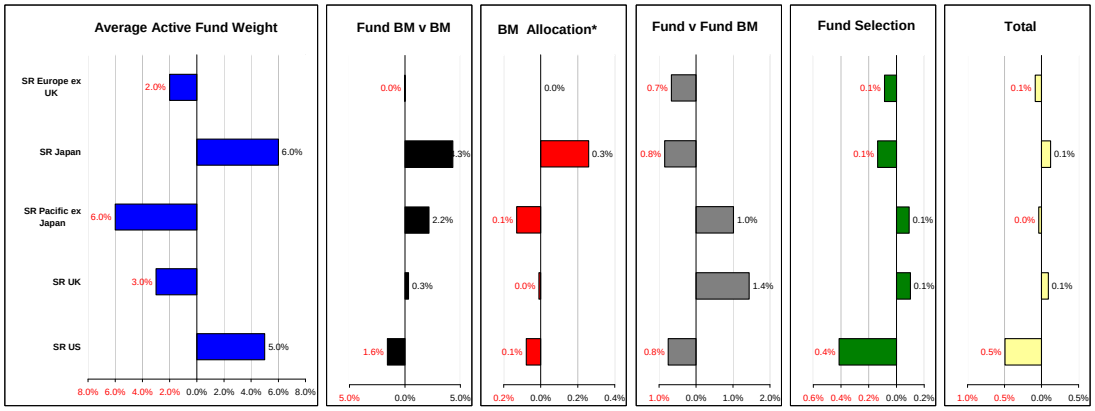
	Returns		
	Portfolio	Benchmark	Active
1 month	-2.4%	-2.0%	-0.4%
3 months	3.4%	3.5%	-0.1%
6 months	7.0%	8.0%	-1.0%
12 months	5.2%	6.2%	-1.0%
Year to Date	0.2%	0.5%	-0.4%
Since Inception	9.4%	9.8%	-0.5%
Since Inception (Annualized)	8.0%	8.4%	-0.4%

The *PerfFund* report, which contains the fund of funds analysis, is shown on the next page. The active return for the fund of funds is decomposed into Allocation and Selection contributions (using the same Brinson methodology as for Market, Sector and Styles). **Allocation** measures how well the manager notionally apportioned the fund to the various fund benchmarks compared with the overall benchmark allocation i.e. assuming the fund weights were applied to the passive benchmarks. In our example, Allocation was a drag on performance over the whole period – the FoF manager's notional allocation to the fund benchmarks was poor. **Selection** measures the impact on the overall fund from each fund's performance against its respective fund benchmark. Selection was positive in this example – overall the individual funds beat their benchmarks. To see how these are calculated, we focus on the most recent month (other periods may be examined using the dropdown menu), where the fund of funds returned -2.4% vs. the benchmark's -2.0%.

Over the latest month, Allocation was a slight positive at +0.04%. This is made up of the relative weights of the funds vs. their benchmarks, times the relative performance of the fund benchmark vs. the overall benchmark. For example, the SR Japan fund has a 16% weight in the fund of funds. Its fund benchmark, MSCI Japan, is 10% of the overall benchmark. Therefore the fund of funds manager has allocated +6% into this fund compared to the benchmark mix (shown as a blue bar in the Contributions by Fund chart). We can think of this as a notional decision to overweight Japan as measured by the MSCI Japan index, so that we implicitly treat the allocation to the Japanese fund as an allocation to the MSCI Japan. From the table, the MSCI Japan index returned +2.3% during the month, and outperformed the overall benchmark by [+2.3% - (-2.0%)] = +4.3% (black bar). So the decision to overweight Japan was a good one, and contributed +6% x 4.3% = 0.26% (red bar). The sum of all these contributions across all funds gives the Allocation return of +0.04%.



Contributions by Fund Last 1 Month (31-Jan-07 to 28-Feb-07)



* Benchmark Allocation has been calculated using active fund weightings.
Allocation = (Fund weight - Fund Benchmark Weight) x (Fund Benchmark Return - Total Benchmark Return)

Fund Returns	1 month	3 months	6 months	12 months	Year to Date	Since Inception	Since Inception (Annualized)
SR Europe ex UK	-2.7%	5.1%	12.4%	17.2%	0.8%	26.4%	22.2%
SR Japan	1.5%	5.7%	2.9%	-1.8%	2.9%	2.1%	1.8%
SR Pacific ex Japan	1.2%	10.1%	18.1%	21.9%	4.9%	32.8%	27.5%
SR UK	-0.2%	5.6%	7.7%	14.8%	2.1%	23.0%	19.4%
SR US	-4.3%	1.0%	5.1%	0.8%	-1.8%	2.6%	2.3%

Fund Benchmark Returns	1 month	3 months	6 months	12 months	Year to Date	Since Inception	Since Inception (Annualized)
FTSE All World Dev Europe (ex UK)	-2.0%	4.5%	11.1%	15.6%	0.4%	22.9%	19.4%
MSCI JAPAN	2.3%	7.9%	5.0%	-3.3%	4.7%	-0.3%	-0.2%
MSCI PACIFIC ex JAPAN	0.2%	7.6%	16.1%	17.3%	2.6%	22.7%	19.1%
FTSE All Share	-1.7%	2.9%	7.2%	12.7%	-0.5%	18.4%	15.6%
S&P 500	-3.6%	1.2%	5.5%	1.0%	-0.7%	2.9%	2.5%

Fund Benchmark Returns	1 month	3 months	6 months	12 months	Year to Date	Since Inception	Since Inception (Annualized)
Composite Benchmark	-2.0%	3.5%	8.0%	6.2%	0.5%	9.8%	8.4%

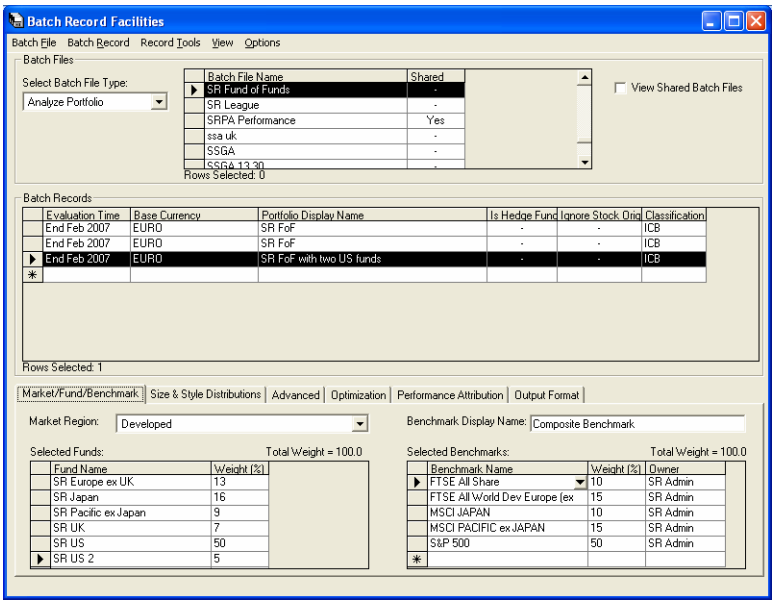
Fund v Fund BM	1 month	3 months	6 months	12 months	Year to Date	Since Inception	Since Inception (Annualized)
FTSE All World Dev Europe (ex UK) (SR Europe ex UK)	-0.7%	0.6%	1.3%	1.6%	0.4%	3.4%	2.9%
MSCI JAPAN (SR Japan)	-0.8%	-2.2%	-2.2%	1.4%	-1.8%	2.4%	2.0%
MSCI PACIFIC ex JAPAN (SR Pacific ex Japan)	1.0%	2.5%	2.1%	4.6%	2.4%	10.1%	8.6%
FTSE All Share (SR UK)	1.4%	2.8%	0.5%	2.1%	2.5%	4.6%	3.9%
S&P 500 (SR US)	-0.8%	-0.2%	-0.4%	-0.2%	-1.1%	-0.3%	-0.2%

Fund BM v Total BM	1 month	3 months	6 months	12 months	Year to Date	Since Inception	Since Inception (Annualized)
FTSE All World Dev Europe (ex UK)	-0.0%	1.0%	3.0%	9.4%	-0.1%	13.1%	11.1%
MSCI JAPAN	4.3%	4.4%	-3.0%	-9.5%	4.2%	-10.1%	-8.7%
MSCI PACIFIC ex JAPAN	2.2%	4.1%	8.0%	11.0%	2.1%	12.8%	10.9%
FTSE All Share	0.3%	-0.6%	-0.9%	6.4%	-1.0%	8.6%	7.3%
S&P 500	-1.6%	-2.3%	-2.5%	-5.2%	-1.2%	-6.9%	-6.0%

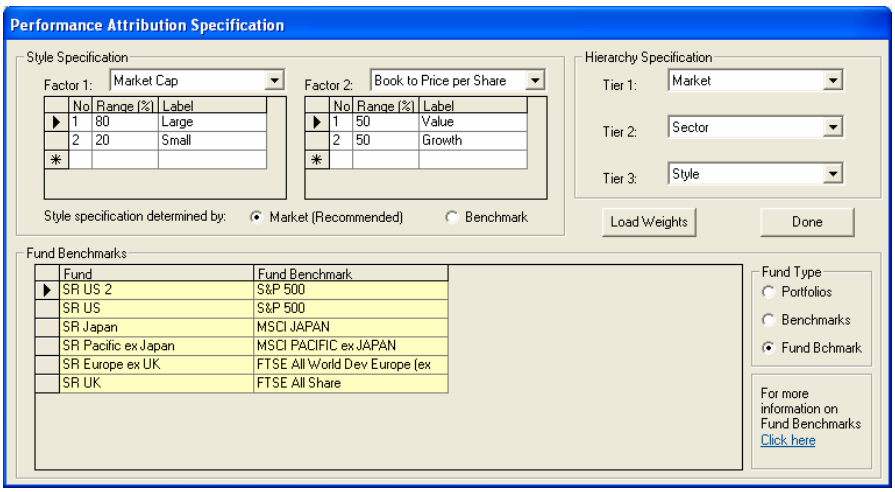
Selection compares the individual fund returns to those of their respective fund benchmarks. In our example, the fund SR Japan had a return of +1.5% over the latest month, which was 0.8% below that of MSCI Japan. This is shown as the grey bar in the chart. Given that 16% of the fund of funds was invested in this fund, the contribution to fund Selection was 16% x (-0.8%) = -0.14% (green bar). Therefore, although the decision to overweight Japan was a good one, the Japanese fund selection impacted negatively. When the Selection contributions are summed across all funds, we get the

Selection return for the month, in this case -0.41% overall. By construction, the Allocation plus the Selection returns add to give the -0.4% that the fund of funds returned relative to the overall benchmark.

A second example is shown below. In this case there are two US funds, *both* associated with the S&P500 fund benchmark. The fund of fund mix for the latest month is shown below:

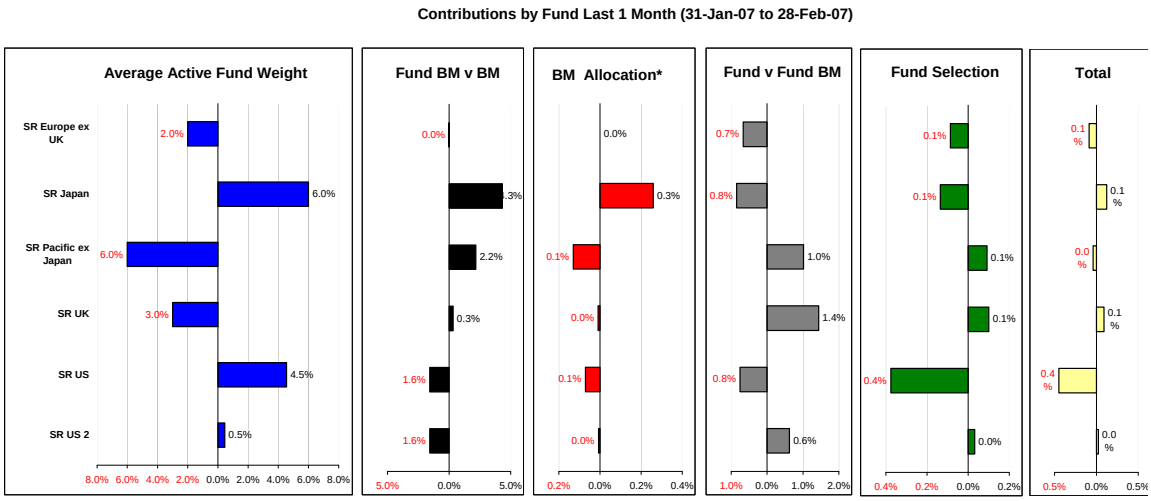


The associations with fund benchmarks are set as:



In this case the total of funds allocated to the S&P 500 is (50% + 5%) = 55%, vs. 50% in the benchmark. The active weight of +5% is divided between the two funds in proportion to their weights in the fund of funds. Consequently, 4.5% of the active position is ascribed to the SR US fund, and 0.5% to SR US 2.

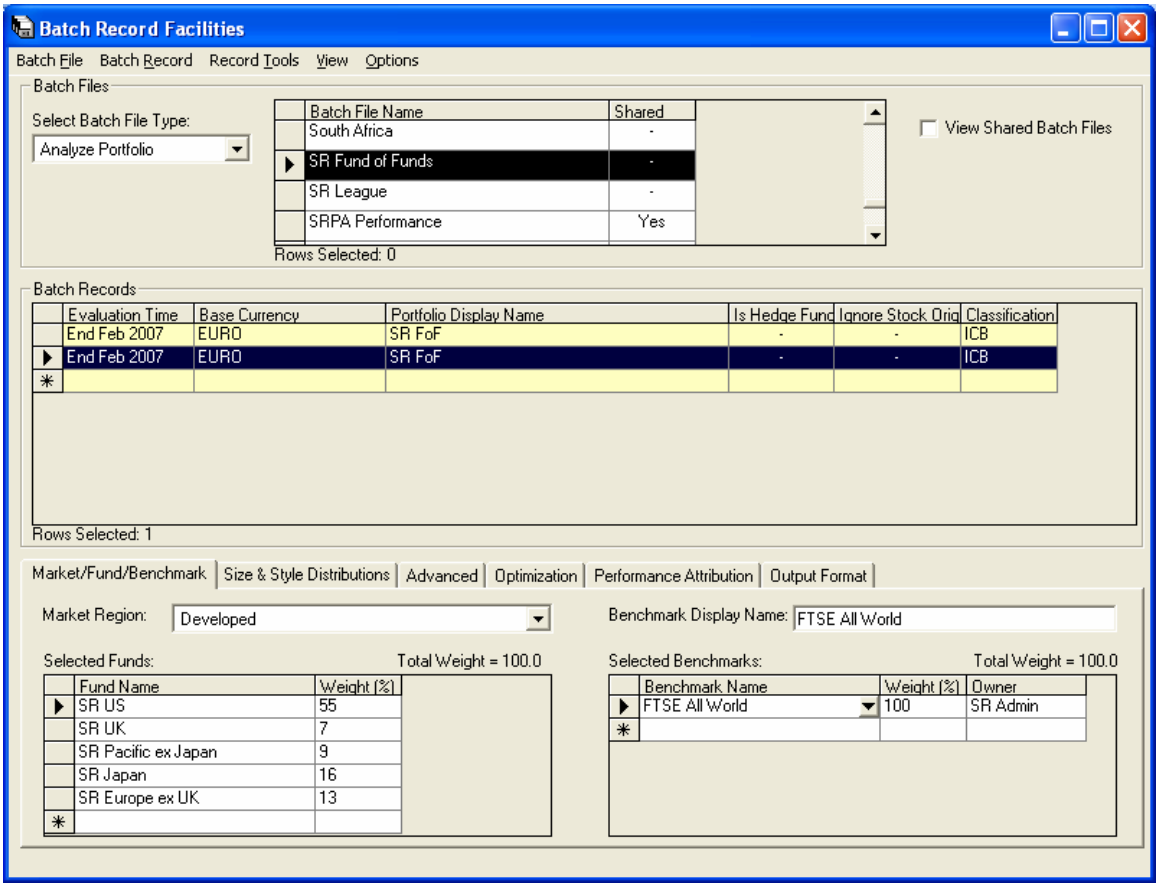
The Contribution by Fund chart will then look like:



All the other calculations are exactly as before. If desired, the Allocation impacts corresponding to the two instances of the S&P500 may be added together, to give the total Allocation impact in relation to that benchmark.

2. Benchmark includes at least one non fund benchmark

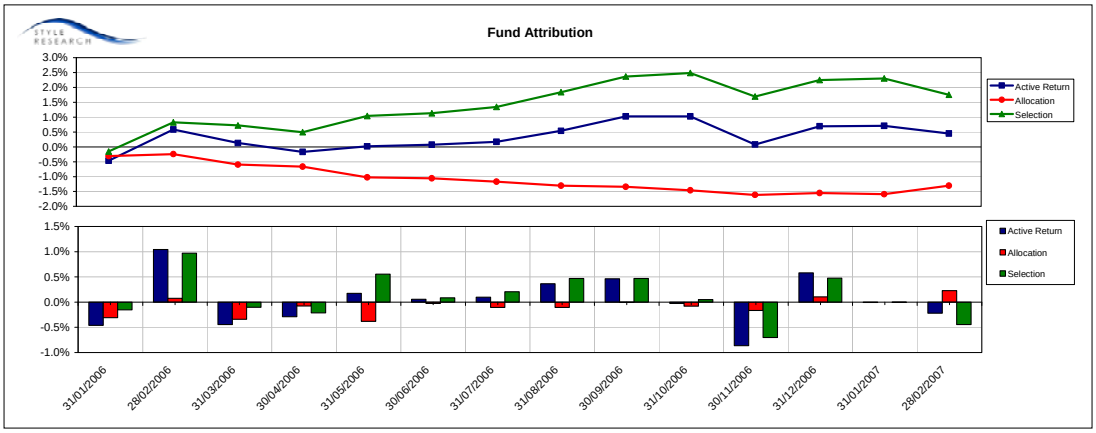
If the benchmark includes at least one non-fund benchmark then it is not possible to express the overall benchmark in terms of fund benchmarks. This is the more general case and would occur if the overall benchmark had an allocation to a benchmark, e.g. MSCI Emerging Markets, which does not have a fund linked to it. Another case might be where the overall benchmark has not been decomposed into sub benchmarks. The example below shows the same fund of funds, but this time simply benchmarked against the FTSE All World Index.



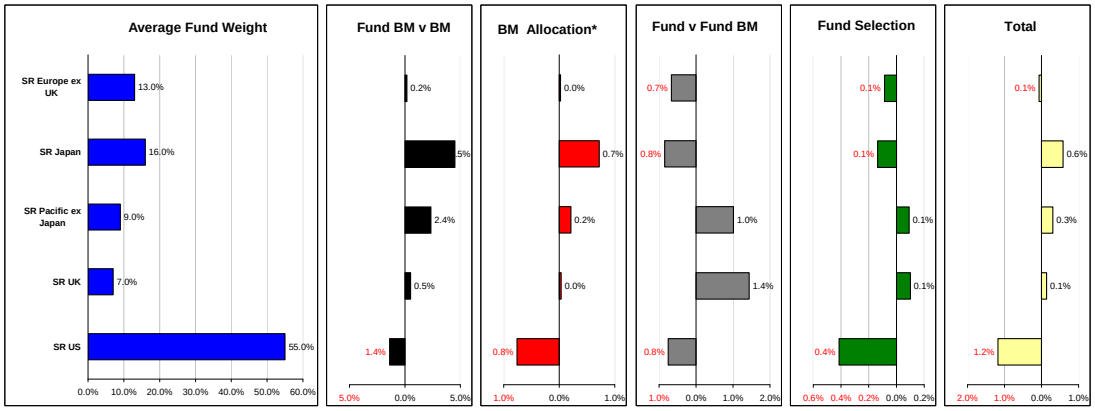
The returns summary is shown below:

	Returns		
	Portfolio	**Benchmark**	**Active**
1 month	-2.4%	-2.2%	-0.2%
3 months	3.4%	3.0%	0.3%
6 months	7.0%	7.1%	-0.1%
12 months	5.2%	5.4%	-0.2%
Year to Date	0.2%	0.4%	-0.2%
Since Inception	9.4%	8.9%	0.5%
Since Inception (Annualized)	8.0%	7.6%	0.4%

Note that the only difference from earlier is that the benchmark return corresponds to the FTSE All World index (-2.2% for the latest month) rather than the previous composite. The funds output is shown on the next page. As before, fund benchmarks *are* associated with each fund, but this time we cannot express the overall benchmark in terms of them. As a consequence we cannot calculate the relative weight of the funds in the fund of funds and the Allocation term is calculated slightly differently. For each fund, Allocation is now the absolute fund weight times its fund benchmark's relative return.



Contributions by Fund Last 1 Month (31-Jan-07 to 28-Feb-07)



* Benchmark Allocation has been calculated using the fund weightings.
Allocation = Fund weight x (Fund Benchmark Return - Total Benchmark Return)

Fund Returns	1 month	3 months	6 months	12 months	Year to Date	Since Inception	Since Inception (Annualized)
SR Europe ex UK	-2.7%	5.1%	12.4%	17.2%	0.8%	26.4%	22.2%
SR Japan	1.5%	5.7%	2.9%	-1.8%	2.9%	2.1%	1.8%
SR Pacific ex Japan	1.2%	10.1%	18.1%	21.9%	4.9%	32.8%	27.5%
SR UK	-0.2%	5.6%	7.7%	14.8%	2.1%	23.0%	19.4%
SR US	-4.3%	1.0%	5.1%	0.8%	-1.8%	2.6%	2.3%

Fund Benchmark Returns	1 month	3 months	6 months	12 months	Year to Date	Since Inception	Since Inception (Annualized)
FTSE All World Dev Europe (ex UK)	-2.0%	4.5%	11.1%	15.6%	0.4%	22.9%	19.4%
MSCI JAPAN	2.3%	7.9%	5.0%	-3.3%	4.7%	-0.3%	-0.2%
MSCI PACIFIC ex JAPAN	0.2%	7.6%	16.1%	17.3%	2.6%	22.7%	19.1%
FTSE All Share	-1.7%	2.9%	7.2%	12.7%	-0.5%	18.4%	15.6%
S&P 500	-3.6%	1.2%	5.5%	1.0%	-0.7%	2.9%	2.5%

Fund Benchmark Returns	1 month	3 months	6 months	12 months	Year to Date	Since Inception	Since Inception (Annualized)
FTSE All World	-2.2%	3.0%	7.1%	5.4%	0.4%	8.9%	7.6%

Fund v Fund BM	1 month	3 months	6 months	12 months	Year to Date	Since Inception	Since Inception (Annualized)
FTSE All World Dev Europe (ex UK) (SR Europe ex UK)	-0.7%	0.6%	1.3%	1.6%	0.4%	3.4%	2.9%
MSCI JAPAN (SR Japan)	-0.8%	-2.2%	-2.2%	1.4%	-1.8%	2.4%	2.0%
MSCI PACIFIC ex JAPAN (SR Pacific ex Japan)	1.0%	2.5%	2.1%	4.6%	2.4%	10.1%	8.6%
FTSE All Share (SR UK)	1.4%	2.8%	0.5%	2.1%	2.5%	4.6%	3.9%
S&P 500 (SR US)	-0.8%	-0.2%	-0.4%	-0.2%	-1.1%	-0.3%	-0.2%

Fund BM v Total BM	1 month	3 months	6 months	12 months	Year to Date	Since Inception	Since Inception (Annualized)
FTSE All World Dev Europe (ex UK)	0.2%	1.4%	3.9%	10.2%	0.0%	14.0%	11.9%
MSCI JAPAN	4.5%	4.8%	-2.1%	-8.6%	4.3%	-9.2%	-7.9%
MSCI PACIFIC ex JAPAN	2.4%	4.5%	8.9%	11.9%	2.2%	13.8%	11.7%
FTSE All Share	0.5%	-0.2%	0.0%	7.3%	-0.9%	9.5%	8.1%
S&P 500	-1.4%	-1.8%	-1.6%	-4.4%	-1.1%	-6.0%	-5.2%

Focusing again on the Japan fund over the latest month, its Allocation contribution is calculated as 16% x (MSCI Japan return less FTSE All World return) = 16% x [2.3% - (-2.2%)] = 0.72%. When this is summed across the contributions from the other funds we get the Allocation return of +0.2% for the period.

Selection is calculated in exactly the same way as before, and gives identical returns in our example (-0.4% over the month). Therefore, the overall active return, made up of Allocation plus Selection, is 0.2% + (-0.4%) = -0.2%.

The monthly data behind all these calculations is listed at the right hand side of the sheet *Agg Perf History*. This data is described in Appendix P1. In common with other performance calculations, the log-linking method is employed over longer time horizons.

The formulae behind these calculations are listed in Appendix P2 (A41-50).

Appendix P1: Navigating the Data Sheets

'Perf Attrib Data' Sheet

This sheet contains more detail behind the Brinson attributions. The first three blocks of data correspond to the three (two, if a single country analysis) different variations by country, sector or Style. The formulae behind these calculations may be found in Appendix P2 – references[8] are provided alongside each item.

For each block of data the following is reported monthly through time. For brevity we describe the market (country) version, but the same elements are found for sector and Style. The first five rows are:

- Allocation – the impact of weighting decisions across all markets– see A29
- Stock Attribution –the impact of stock selection within markets across all markets – see A31
- Interaction – the interplay between allocation and selection – see A33
- Cash Attribution – the allocation component due to cash holdings – see A29
- Currency Attribution – the impact of active currency bets – see A21

At any point in time these components will sum to the active base currency return for that month. The next five rows are the cumulative versions of the above. At any point in time they will add to give the cumulative active base currency return from the start date to that time. These are followed by the high level returns, listed below:

- Portfolio return in base currency – see A5
- Benchmark return in base currency – A6
- Active return in base currency – the difference between A5 and A6
- Portfolio return in local currency – see A3
- Benchmark return in local currency – see A4
- Active return in local currency – the difference between A3 and A4

The next six items are the cumulative versions of the above.

Following this are two analogous blocks of data corresponding to sector and Style. After these there are three further blocks of data where the elements of the Brinson calculations are laid out for the latest month. This data may also be found, for all dates, in the *Agg Perf History* sheet. Once again focusing on the market version, the data displayed contains the following for each market. Starting with the leftmost column, the data shown are:

- Portfolio weight in market
- Benchmark weight in market
- Active weight in market

[8] e.g. A29 refers to the formula found in Appendix P2 under that reference.

- Portfolio return in local currency for market – see A23
- Benchmark return in local currency for market – see A24
- Benchmark local currency return for country relative to entire benchmark local currency return – second bracket of A29 or equivalently A29 divided by the active weight for that country.
- Allocation – the impact of market decisions across all markets– see A29
- Stock Attribution –the impact of stock selection within markets across all markets – see A31
- Interaction – the interplay between allocation and selection– see A33
- Total effect – sum of allocation, selection and interaction. Per market this is not particularly meaningful. However, summed across all markets (including cash) it equates to the overall local currency active return for the latest month.

The next two blocks of data report the analogous items for the sector and Style Brinson cases.

'Agg Perf History' Sheet

This sheet reports the detailed contributing elements for the Brinson calculations for each month. The order of the links closely reflects the formulae shown in the Brinson section of Appendix P2.

There are up to five sets of data, each with embedded links at the top of the spreadsheet. As usual, the first three (where relevant) relate to the Brinson decompositions, by country, sector and Style. The fourth is an additional breakdown, by currency. This shows the influence of currency on performance. Finally, a fifth set corresponds to the fund of funds report analysis if selected.

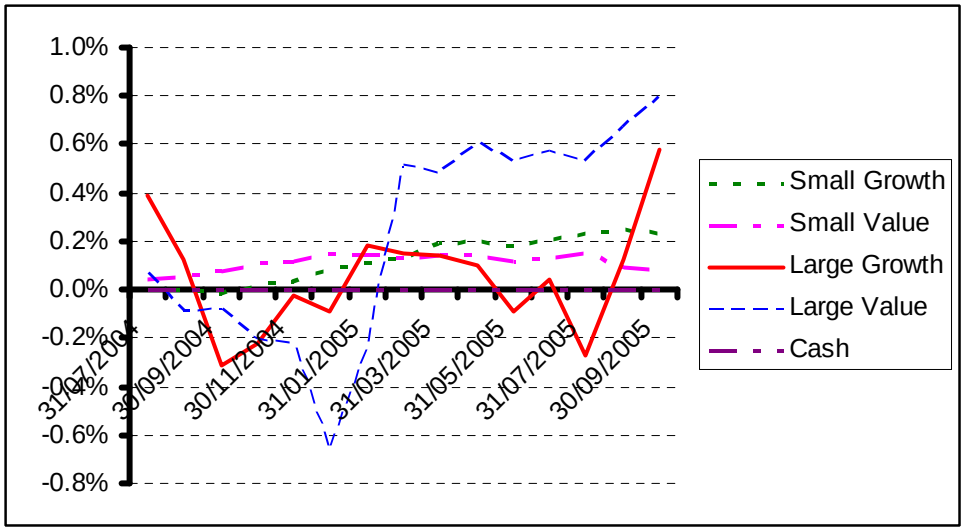
The first fifteen data sets for any of the three Brinson decompositions are described in sections 1-15 of Appendix P2 and are therefore not repeated here. The next fifteen are the cumulative versions of the latter – see Appendix P3 for the methodology employed in the chain-linking process. Note that where appropriate, P2 explains the sum of the cross sectional components.

Following this are data on the weights per category for the portfolio, benchmark and active positions, and then the main Brinson allocation, selection and interaction components, per month and cumulatively. These correspond to sections 16-18 of Appendix P2.

The final set of 'reverse cumulative' data are mainly used for charting purposes. They show the cumulative return at any month end corresponding to the cumulative contribution *from* that month *to* the latest date. For example, the data for allocation for month *Dec 2004* would correspond to the allocation contribution from *Dec 2004* to the latest date.

In addition to the three Brinson data sets, there is also per month and cumulative information per currency and per fund. See the formulae in sections 19-21 of Appendix P2 for the currency formulae and in sections 22 through 32.

This data is particularly useful for the creation of custom charts. For example, we saw that stock selection was reasonably good for the case study, whether the analysis was examined by market, sector or Style. What did the components of this selection look like when split according to stocks in, say, different Styles? In order to answer this question, the data may be located quickly by clicking on the 'Cum. Contribution to Stock Selection' by Style link. This block of data may be charted immediately, using the Excel chart wizard, to quickly reveal a graph such as the following:



It shows that stock selection has been quite volatile within large cap, for both Value and Growth. Within small caps performance has been fairly moderate and consistent through time.

This is one of many similar charts that could be generated very easily. Stock selection within other groupings, allocation impacts, weights and returns can all be accessed in this sheet.

'Perf Summary Data' Sheet

This sheet contains a summary of portfolio and benchmark return information, per month and cumulatively.

The information is described per row. Please see the Brinson formulae section In Appendix P2 for the precise calculations.

- Portfolio Return (Base Curr) – see A5 or A14
- Benchmark Return (Base Curr) – see A6 or A16
- Active Return (Base Curr) – difference between above, or A18
- Portfolio Return (Local Curr) – see A3 or A8
- Benchmark Return (Local Curr) – see A4 or A10
- Active Return (Local Curr) – difference between above, or A12

- Portfolio Currency Return – A36 summed over all currencies
- Benchmark Currency Return – A37 summed over all currencies
- Active Currency Return – A38 summed over all currencies
- Portfolio Cash Return – Cash part of A7
- Benchmark Cash Return – Cash part of A9
- Active Cash Return – Cash part of A11
- Portfolio Cash Currency Return – Cash part of A36
- Benchmark Cash Currency Return – Cash part of A37
- Active Cash Currency Return – Cash part of A38
- Portfolio Equity Return – Non-cash (i.e. equity) part of A8
- Benchmark Equity Return – Non-cash (i.e. equity) part of A10
- Active Equity Return – Non-cash (i.e. equity) part of A12
- Portfolio Equity Currency Return – Non-cash part of A36
- Benchmark Equity Currency Return – Non-cash part of A37
- Active Equity Currency Return – Non-cash part of A38

The next 21 rows correspond to the cumulative versions of the above. Please see the section on chain-linking returns in Appendix P3 for the methodology employed.

The Turnover number is the (one-sided) month to month change in holdings. The constituents of the portfolio as of month end *t-1* are run forward to the end of month *t* using the returns for that month. These are then compared with the holdings at month end *t* and the differences between the holdings are calculated. The absolute values of these differences are then added and divided by two. This is equivalent to the notional buys (or sells) required to rebalance the *t-1* portfolio to that of time *t*, undertaken at time *t.*

The remaining rows report the actual portfolio return (if loaded) and the trading return. The actual portfolio return is intended to be an independently calculated fully reconciled transactions-based, time-weighted portfolio return. This is compared with the buy and hold portfolio return calculated using the supplied monthly holdings. The difference, reported as the trading return, reveals the degree to which the buy-and-hold returns are similar to the 'official' transactions based time-weighted returns.

The last block of data (starting around row 110) is the underlying data for the **Top/Bottom 10 Contributors to Active Return** shown in the chart at the bottom of *Perf Summary*. There are several sets of data corresponding to the different periods that the user can select in the spreadsheet.

- The average active weight is the sum of the month by month active weights for a given stock divided by the number of months in the relevant period (or the number of months that the stock has existed, if a new issue).

- The relative return is the difference between the (cumulative, if appropriate) return of the stock and the benchmark return over the period, in base currency terms.

- Active Return Contribution (on which the top/bottom ten is based) is the stock by stock active return over the selected period. Each month, for every stock, the active weight times the active return is calculated. These contributions are then adjusted, using a standard chain-linking methodology, so that over the appropriate period the cumulative active return equals the difference in cumulative portfolio less cumulative benchmark return. In other words, if the entire list of stocks were shown (as opposed to just the top/bottom ten) the contributions would sum to give the active cumulative base currency return for the selected period.

'Perf Top 10 Contributors' Sheet

This data is used in the **Top/Bottom 10 Contributions** charts within the three Brinson analyses. There is one set for market (if multi-country), one for sector and one for Style.

Within each grouping there are three pieces of data:

- The Active Weight of each stock is *within* the appropriate group. For example, within markets it would correspond to the portfolio stock weight as a percentage of the portfolio's weight within that stock's country, less the corresponding (within country) weight for the stock in the benchmark. Using the notation set out in Appendix P1, for stock *i*, a member of group (e.g. market) *G*, this is: $w_i^P / W_G^P - w_i^B / W_G^B$.

- The next column is the local currency return, r_i^l, of the stock over the selected period, relative to the local currency return of its group (e.g. market).

- The final column is the contribution per stock to the Brinson stock selection return for the appropriate group. It consists of the stock by stock contributions to A31. Essentially, each of these reduces to the (within group) active weight of the stock times the stock's local return relative to its local group return, multiplied by the benchmark weight in that stock's group. Adding all contributions across the stocks in a group gives the stock selection contribution for that group.

'SR Perf Attrib Data' Sheet

This sheet contains all the background data for the hierarchical approach to performance attribution. The output will clearly depend on the user choices of hierarchy and Style definitions. However, we focus here on the default settings as the general case.

The first block of data shows the main five components of the hierarchical breakdown, monthly and cumulatively. The returns are based on the formulae laid out in Appendix P1 and correspond to the terms in equation B11. A standard chain-linking approach is employed so that the cumulative data at any point in time will sum to give the cumulative active return at that date.

The second block sets out the hierarchical data in a panel with three Excel filters. This provides a means of isolating the hierarchical return components for different groups of stocks. It is a dynamic way of examining returns analogous to the risk data found in the '*Risk Decomposition*' sheet of a regular Style Research Portfolio Analyzer analysis. For example, a user interested in seeing the impact of stock selection within Large Growth companies, could use this table. The three filters are:

1. The Attribution Type – i.e. which element of equation B11 is being reported.
2. The Group Type – whether to examine the data by market, sector, Style or currency
3. Group – the specific group (country/sector/Style/currency) required within the Group Type.

The use of this table is best illustrated with a few examples.

Example I

Attribution Type – *unfiltered*
Group Type – *Market*
Group – *Japan*

These settings list the five hierarchical components (of B11), but aggregated by market, so that the contributions for currency, market, sector, Style and stock are shown for the Japanese market. Here's an extract of this filter for the first three months of the case study:

Attribution Type	Group Type	Group	31/07/2004	31/08/2004	30/09/2004
Currency	Market	Japan	-0.1%	0.0%	0.0%
Market	Market	Japan	0.0%	0.0%	-0.1%
Sector	Market	Japan	0.2%	-0.1%	0.0%
Style	Market	Japan	0.0%	0.0%	0.0%
Stock	Market	Japan	-0.2%	-0.1%	-0.2%
Cum Currency	Market	Japan	-0.1%	0.0%	0.0%
Cum Market	Market	Japan	0.0%	-0.1%	-0.1%
Cum Sector	Market	Japan	0.2%	0.1%	0.2%
Cum Style	Market	Japan	0.0%	0.0%	0.0%
Cum Stock	Market	Japan	-0.2%	-0.3%	-0.5%

Sector selection OK

Stock selection weak

It shows, for example, that stock picking was weak in Japan over this period, though sector allocation within Japan was reasonable.

Example II

Attribution Type – *Cumulative Stock Selection*
Group Type – *Style*
Group – *Large Growth*

In this case the data have been narrowed down so that just the cumulative Stock return component of B11 is reported. The filters on Group Type and Group focus the results further so that only contributions from stock selection within Large Growth stocks are considered. Here's the data for the last three months of the analysis period.

Attribution Type	Group Type	Group	31/07/2005	31/08/2005	30/09/2005
Cum Stock	Style	Large Growth	-0.4%	-0.2%	-0.4%

It shows that stock selection within Large Growth stocks was weak over the period. Given that this breakdown closely mirrors the risk decomposition, it is possible to compare the risk taken in this area with the return delivered. Note that this information was previously uncovered in the pivot table discussion. But the data presented here goes further, in that it shows the data month by month as well as cumulatively.

Example III

Attribution Type – *Sector*
Group Type – *Sector*
Group – *Unfiltered*

In the main hierarchical report, *SR Perf Chart*, the attribution revealed that sector allocation within countries contributed 0.4% to performance in the first month of the analysis. The above settings reveal the breakdown of this contribution across sectors:

Attribution Type	Group Type	Group	31/07/2004
Sector	Sector	Financials	0.1%
Sector	Sector	Energy	0.1%
Sector	Sector	Utilities	0.0%
Sector	Sector	Information Tec	0.0%
Sector	Sector	Health Care	0.2%
Sector	Sector	Consumer Stap	0.0%
Sector	Sector	Consumer Disc	0.1%
Sector	Sector	Telecommunica	0.0%
Sector	Sector	Industrials	0.0%
Sector	Sector	Materials	0.0%
Sector	Sector	No Sector	0.0%
Sector	Sector	Cash	0.0%

In this case it can be seen that Health Care was amongst the best bets during this month. Note that these returns sum to the 0.4%.

Clearly there are many variations of the above settings which will allow the user to find the returns corresponding to specific hierarchical components, dates and groups.

There is a special case corresponding to the choice of Group Type = '*Currency*'. When this is selected the active return is broken up by currency, reported either monthly or cumulatively.

The third block of data in *SR Perf Attrib Data* contains the data behind the pivot table in *SR Perf Chart*. As such it is best interrogated using the pivot table. For a variety of periods it shows each of the five hierarchical components (B11) for every market/sector/Style combination.

'Perf Stock Level Data' Sheet

This sheet contains the detailed per stock data for the dates requested by the user (default is data over the latest 3 months). As such, it has the potential to be very large as it contains data on every stock in the union of the portfolio and the benchmark. For example, a three year analysis where the benchmark is the Russell 2000, would contain 2000 times 36 rows = 72,000 rows, which exceeds the number of rows allowed in an Excel spreadsheet. In such cases, the data is truncated to the number of complete months that will fit into the sheet (so 32 months would be the limit for the Russell 2000 – the earlier months would have to be generated in a separate run).

The best way to navigate the data in this sheet is to use the Excel Autofilter feature. Clicking on the various dropdowns enables the user to examine specific data at the stock level. For reference, here are the definitions of the columns. Note that this description assumes that all three Brinson and the hierarchical runs have been undertaken for a multi-country portfolio. Each row corresponds to a unique stock and date combination.

Column	**Description**	**Formula Ref/Comment**
A	Worldscope Stock Code	-
B	Date	*t where shown explicitly*
C	Stock Name	-
D	Currency of domicile	-
E	Market of domicile	-
F	Sector	-
G	Style (Brinson)	*uses pan-region Styles*
H	Style (Hierarchical)	*e.g. Style within sector within country*
I	Stock weight in benchmark[9]	$w_i^B(t)$
J	Stock weight in portfolio	$w_i^P(t)$
K	Stock active weight	$a_i(t)$
L	Stock return in local currency[9]	$r_i^l(t)$
M	Stock return in base currency	$r_i^{base}(t)$
N	B'mark rel stock return in base curr.	$r_i^{base}(t) - R^{Bbase}(t)$
O	Contrib. to active base curr. return	$a_i(t) * [r_i^{base}(t) - R^{Bbase}(t)]$
P	Currency return	see A1
Q	B'mark rel. currency return	$[r_i^{base} - R^{Bbase}] - [r_i^l - R^{Bl}]$ (*cf.* B2)
R	Market component	see B3
S	Sector component	see B5
T	Style component	see B7
U	Stock component	see B9
V	Currency Attribution	B11(a)
W	Market Attribution	B11(b)

[9] Note the convention laid out in the Notation section of Appendix P1. Weights refer to start month weights (or equivalently the weights as at the *previous* month-end. So if column B refers to Sep 2005, all weights correspond to those at the end of Aug 2005, and all returns are from end Aug 2005 to end Sep 2005.

X	Sector Attribution	B11(c)
Y	Style Attribution	B11(d)
Z	Stock Attribution	B11(e)
AA	Contrib. Currency Return (Brins)	$a_i(t)[r_i^{base}(t) - r_i^l(t)]$ see A21

Col.	**Description**	**Formula Ref/Comment**
AB	Contribution to Stock Selection (Brinson Market)	
	$W_{Market}^B(w_i^P / W_{Market}^P - w_i^B / W_{Market}^B)(r_i^l - R_{Market}^{Bl})$	see A31
AC	Contribution to Stock Selection (Brinson Sector)	
	$W_{Sector}^B(w_i^P / W_{Sector}^P - w_i^B / W_{Sector}^B)(r_i^l - R_{Sector}^{Bl})$	see A31
AC	Contribution to Stock Selection (Brinson Style)	
	$W_{Style}^B(w_i^P / W_{Style}^P - w_i^B / W_{Style}^B)(r_i^l - R_{Style}^{Bl})$	see A31
AE	Active weight in Market (Brinson)	$(w_i^P / W_{Market}^P - w_i^B / W_{Market}^B)$
AF	Active weight in Sector (Brinson)	$(w_i^P / W_{Sector}^P - w_i^B / W_{Sector}^B)$
AG	Active weight in Style (Brinson)	$(w_i^P / W_{Style}^P - w_i^B / W_{Style}^B)$
AH	Market relative stock return	$(r_i^l - R_{Market}^{Bl})$
AI	Sector relative stock return	$(r_i^l - R_{Sector}^{Bl})$
AJ	Style relative stock return	$(r_i^l - R_{Style}^{Bl})$

This data is useful for viewing stock level contributions according to various attributions. It is possible, using this information, to reproduce any of the data on the preceding sheets. But the primary use for this sheet is to give the user the ability to dig into areas of interest for a particular analysis by using the 'Autofilter' on the data and filtering accordingly.

A good starting point is to filter by date. For our case study, stock selection within Styles (Brinson, as displayed in *PerfStyle*) added 0.4% in Sep 2005. By filtering on that date, column AD (Contribution to Stock Selection by Style) sums to the same 0.4%. The column may be sorted and the explicit stock by stock contributors analyzed. At this stage a further filter could be added, such as selecting only Large Growth stocks. Following this, the filtered stocks in column AD will now add to the Large Growth stock contribution to stock selection as of Sep 2005. Of course other filters could be applied, such as sector and market.

Columns Q through Z refer to the hierarchical breakdown. If the data is filtered on a specific date (e.g. Sep 2005), columns V through Z will add to give the overall active return. Individually the columns add to their respective hierarchical contributions. In our case study, column X will add to give -0.05%, the sector level contribution for Sep 2005. Sorting on this column will then show the biggest contribution stock by stock.

Once individual stocks are identified, the information on their returns, active weights or weights within market/sector/Style may be examined to give a detailed bottom up understanding of what has been the source of the higher level attribution data.

Columns AH through AJ are the local currency stock returns relative to their appropriate group returns for the benchmark. They are employed in the **Top/Bottom 10 Contributors Within Market/Sector/Style** charts at the bottom of the Brinson based *PerfMarket, PerfSector* and *PerfStyle* sheets respectively.

Appendix P2: Detailed Calculations

Notation

The conventions used throughout the formulae definitions are listed below.

Weights

$w_i^P(t)$ is the weight of stock *i* in the portfolio at the beginning of month *t*
$w_i^B(t)$ is the weight of stock *i* in the benchmark at the beginning of month *t*
$a_i(t) = w_i^P - w_i^B$ is the active weight of stock *i* at the beginning of month *t*

e.g., if *t = September 2005*, the weights refer to the beginning of *September 2005* weights (or equivalently the end of *August 2005* weights)

For the Brinson decomposition, the region is divided into countries or sectors or Styles. Weights, at a Group level (where Groups could be countries *or* sectors *or* Styles), are as follows:

$W_G^P(t) = \sum_{i \in G} w_i^P(t)$ is the weight of group G in the Portfolio at the beginning of month *t*

$W_G^B(t) = \sum_{i \in G} w_i^B(t)$ is the weight of group G in the Benchmark at the beginning of month *t*

e.g., $W_{Japan}^B(t) = \sum_{i \in Japan} w_i^B(t)$ is the weight of Japan in the Benchmark at the beginning of month *t*

Returns

All returns are total returns.

$r_i^{base}(t)$ is the base currency return of stock *i* over month *t*
$r_i^l(t)$ is the local currency return of stock *i* over month *t*
e.g., if *t = Sep 2005*, the returns are for the period *end Aug 2005* to *end Sep 2005*.
If the exchange rate between base and local currency *l* is $X^l(t)$ (in units of base per local) at the end of month *t*, the exchange rate *return* for currency *l*, over that month, is defined as:

$$RX^l(t) = X^l(t) / X^l(t-1) - 1 \qquad \text{(A1)}$$

Thus, by definition, $(1 + r_i^{base}(t)) = (1 + RX^l(t)) * (1 + r_i^l(t))$ (A2)

Though it is not stated explicitly, remember that *l* will refer to the local currency corresponding to the domicile of stock *i*.

We are now in a position to present the formulae behind the information shown in the data sheets and charts. All references are to time *(t)*, so for brevity this has been dropped from the formulae.

In particular, note that:

$$R^{Pl} = \sum_{\forall i} w_i^P r_i^l \quad \text{is the Portfolio return in local currency terms,} \tag{A3}$$

$$R^{Bl} = \sum_{\forall i} w_i^B r_i^l \quad \text{is the Benchmark return in local currency terms,} \tag{A4}$$

$$R^{Pbase} = \sum_{\forall i} w_i^P r_i^{base} \quad \text{is the Portfolio return in base currency terms,} \tag{A5}$$

$$R^{Bbase} = \sum_{\forall i} w_i^B r_i^{base} \quad \text{is the Benchmark return in base currency terms.} \tag{A6}$$

Active returns are denoted as follows:

$$R^{abase} = \sum_{\forall i} \left(w_i^P - w_i^B \right) r_i^{base}$$

Brinson Formulae

1. Portfolio Return Attribution (Local)

For group G,

$$RA_G^{Pl} = \sum_{i \in G} w_i^P r_i^l \tag{A7}$$

This information may be found in sheet '*Agg Perf History*' under 'Portfolio Return Attribution (Local)' [10].

Note that this sums across all groups to the local currency portfolio return, i.e.:

$$R^{Pl} = \sum_{\forall G} RA_G^{Pl} \tag{A8}$$

[10] Note that this does <u>not</u> equal the return of the portfolio in that group (it needs to be scaled by the weight of the portfolio in that group – see Portfolio Return (Local)).

2. Benchmark Return Attribution (Local)

For group G,

$$RA_G^{Bl} = \sum_{i \in G} w_i^B \, r_i^l \tag{A9}$$

This information may be found in sheet '*Agg Perf History*' under 'Benchmark Return Attribution (Local)'.

Note that this sums across all groups to the local currency benchmark return, i.e.:

$$R^{Bl} = \sum_{\forall G} RA_G^{Bl} \tag{A10}$$

3. Active Return Attribution (Local)

For group G,

$$RA_G^{al} = \sum_{i \in G} a_i \, r_i^l = \sum_{i \in G} \left(w_i^P - w_i^B \right) r_i^l \tag{A11}$$

This information may be found in sheet '*Agg Perf History*' under 'Active Return Attribution (Local)'.

Note that this sums across all groups to the local active return, i.e.:

$$R^{al} = \sum_{\forall G} RA_G^{al} \tag{A12}$$

4. Portfolio Return Attribution (Base)

For group G,

$$RA_G^{Pbase} = \sum_{i \in G} w_i^P \, r_i^{base} \tag{A13}$$

This information may be found in sheet '*Agg Perf History*' under 'Portfolio Return Attribution (Base)'.

Note that this sums across all groups to the base currency portfolio return, i.e.:

$$R^{Pbase} = \sum_{\forall G} RA_G^{Pbase} \tag{A14}$$

5. Benchmark Return Attribution (Base)

For group G,

$$RA_G^{Bbase} = \sum_{i \in G} w_i^B \, r_i^{base} \tag{A15}$$

This information may be found in sheet '*Agg Perf History*' under 'Benchmark Return Attribution (Base)'.

Note that this sums across all groups to the base currency benchmark return, i.e.:

$$R^{Bbase} = \sum_{\forall G} RA_G^{Bbase} \tag{A16}$$

6. Active Return Attribution (Base)

For group G,

$$RA_G^{aBase} = \sum_{i \in G} a_i \, r_i^{base} = \sum_{i \in G} \left(w_i^P - w_i^B \right) r_i^{base} \tag{A17}$$

This information may be found in sheet '*Agg Perf History*' under 'Active Return Attribution (Local)'.

Note that this sums across all groups to the base currency active return, i.e.:

$$R^{aBase} = \sum_{\forall G} RA_G^{aBase} \tag{A18}$$

7. Portfolio Currency Return Attribution

We define, for group G, the Portfolio Currency Return Attribution as:

$$FX_G^P = \sum_{i \in G} w_i^P \left(r_i^{base} - r_i^l \right) \tag{A19}$$

This information may be found in sheet '*Agg Perf History*' under 'Portfolio Currency Return Attribution'.

8. Benchmark Currency Return Attribution

We define, for group G, the Benchmark Currency Return Attribution as:

$$FX_G^B = \sum_{i \in G} w_i^B \left(r_i^{base} - r_i^l \right) \tag{A20}$$

This information may be found in sheet '*Agg Perf History*' under 'Benchmark Currency Return Attribution'.

9. Active Currency Return Attribution

We define, for group G, the Active Currency Return Attribution as:

$$FX_G^a = \sum_{i \in G} a_i \left(r_i^{Base} - r_i^l \right) = \sum_{i \in G} \left(w_i^P - w_i^B \right) \left(r_i^{Base} - r_i^l \right) \qquad \text{(A21)}$$

This information may be found in sheet '*Agg Perf History*' under 'Active Currency Return Attribution'.

In the special case where G represents *countries* and consequently all the stocks in a group have the same local currency, L say, it is easy to show that this is equivalent to:

$$FX_G^a = RX^L [(W_G^P - W_G^B) + RA_G^{aL}] \qquad \text{(A22)}$$

Or, in words, the currency return attribution for *country* G is the exchange rate return for that country scaled by the active position plus the local currency return attribution of that country.

10. Portfolio Return (Local)

For group G,

$$R_G^{Pl} = \frac{\sum_{i \in G} w_i^P r_i^l}{\sum_{i \in G} w_i^P} = \frac{RA_G^{Pl}}{W_G^P} \qquad \text{(A23)}$$

This information may be found in sheet '*Agg Perf History*' under 'Portfolio Return (Local)'.

11. Benchmark Return (Local)

For group G,

$$R_G^{Bl} = \frac{\sum_{i \in G} w_i^B r_i^l}{\sum_{i \in G} w_i^B} = \frac{RA_G^{Bl}}{W_G^B} \qquad \text{(A24)}$$

This information may be found in sheet '*Agg Perf History*' under 'Benchmark Return (Local)'.

12. Active Return (Local)

For group G,

$$R_G^{al} = R_G^{Pl} - R_G^{Bl} = \frac{\sum_{i \in G} w_i^P r_i^l}{\sum_{i \in G} w_i^P} - \frac{\sum_{i \in G} w_i^B r_i^l}{\sum_{i \in G} w_i^B} = \frac{RA_G^{Pl}}{W_G^P} - \frac{RA_G^{Bl}}{W_G^B} \quad \text{(A25)}$$

This information may be found in sheet '*Agg Perf History*' under 'Active Return (Local)'.

13. Portfolio Return (Base)

For group G,

$$R_G^{Pbase} = \frac{\sum_{i \in G} w_i^P r_i^{base}}{\sum_{i \in G} w_i^P} = \frac{RA_G^{Pbase}}{W_G^P} \quad \text{(A26)}$$

This information may be found in sheet '*Agg Perf History*' under 'Portfolio Return (Base)'.

14. Benchmark Return (Base)

For group G,

$$R_G^{Bbase} = \frac{\sum_{i \in G} w_i^B r_i^{base}}{\sum_{i \in G} w_i^B} = \frac{RA_G^{Bbase}}{W_G^B} \quad \text{(A27)}$$

This information may be found in sheet '*Agg Perf History*' under 'Benchmark Return (Base)'.

15. Active Return (Base)

For group G,

$$R_G^{abase} = R_G^{Pbase} - R_G^{Bbase} = \frac{\sum_{i \in G} w_i^P r_i^{base}}{\sum_{i \in G} w_i^P} - \frac{\sum_{i \in G} w_i^B r_i^{base}}{\sum_{i \in G} w_i^B} = \frac{RA_G^{Pbase}}{W_G^P} - \frac{RA_G^{Bbase}}{W_G^B} \quad \text{(A28)}$$

This information may be found in sheet '*Agg Perf History'* under 'Active Return (Base)'.

The next fifteen sets of data in '*Agg Perf History*' are the cumulative versions of the above – see Appendix P3 for details.

We are now in a position to list formulae for the Brinson components

16. Allocation Contribution

For group G,

$$R_G^{Allocation} = \left(W_G^P - W_G^B\right)\left(R_G^{Bl} - R^{Bl}\right) \qquad \text{(A29)}$$

In words, it is the active weight of the portfolio in group G times the local currency return of G relative to the local currency benchmark return. Thus, an overweight allocation to a segment of the market that has outperformed the benchmark, will give rise to a positive contribution from allocation.

Summed across all groups, this gives the total Allocation Contribution at time *t*:

$$R^{Allocation} = \sum_{\forall G} R_G^{Allocation} \qquad \text{(A30)}$$

This information may be found in sheet '*Agg Perf History'* under 'Contribution from Allocation'

17. Stock Selection Contribution

For group G,

$$R_G^{Selection} = W_G^B\left(R_G^{Pl} - R_G^{Bl}\right) \qquad \text{(A31)}$$

The Stock Selection Contribution for group G is therefore the difference between portfolio and benchmark returns in that group, weighted by the benchmark weight for group G. Consequently, it will be positive if the portfolio has outperformed the benchmark in a particular group.

Summed across all groups, this gives the total Stock Selection Contribution at time *t*:

$$R^{Selection} = \sum_{\forall G} R_G^{Selection} \qquad \text{(A32)}$$

This information may be found in sheet '*Agg Perf History'* under 'Contribution from Stock Selection'

18. Interaction Contribution

For group G,

$$R_G^{Interaction} = \left(W_G^P - W_G^B\right)\left(R_G^{Pl} - R_G^{Bl}\right) \qquad \text{(A33)}$$

The Interaction Contribution for group G, is therefore the return difference between portfolio and benchmark scaled by the active weight for that group. It will be positive when, for example, there is an active overweight to a group where the portfolio has outperformed the benchmark.

Summed across all groups, this gives the total Interaction Contribution at time *t*:

$$R^{Interaction} = \sum_{\forall G} R_G^{Interaction} \qquad \text{(A34)}$$

This information may be found in sheet '*Agg Perf History'* under 'Contribution from Interaction'.

It is easy to show that the active local return of the Portfolio vs. the Benchmark is:

$$R^{al} = R^{Allocation} + R^{Selection} + R^{Interaction} = \sum_{\forall G} (R_G^{Allocation} + R_G^{Selection} + R_G^{Interaction}) \qquad \text{(A35)}$$

Thus the Brinson components add back to the active local return, as required. Remember that these formulae apply to countries, sectors and Styles separately, and the data may be quickly referenced using the links at the top of the first three columns in sheet '*Agg Perf History'*. Note that cash is shown as a component of allocation within '*Agg Perf History'*. It is separated out when reported in '*Perf Attrib Data'*.

An alternative version of the Brinson Stock Selection contribution is sometimes used by the performance analysis community. This uses the *portfolio weighted* relative return within a group, i.e.:

$$R_G^{PortfolioWeightedSelection} = W_G^P \left(R_G^{Pl} - R_G^{Bl} \right) \qquad \text{(A31b)}$$

An appealing property of this is that its use mitigates the need for an interaction term. It is easy to show that:

$$R^{al} = R^{Allocation} + R^{PortfolioWeightedSelection}$$

However, the disadvantage of this definition compared with the benchmark weighted version is that it combines stock selection within a group with the amount allocated to that group. In practice these may be separate decisions. Using the benchmark weights is more *neutral* in this respect and hence we have chosen this method to provide a clearer separation of stock selection with group allocation. It is easy to show that the Stock Selection Contribution (A31) plus the Interaction Contribution (A33), add to give the *portfolio weighted* Stock Selection (A31b).

There is some additional currency data in the fourth column of that sheet, as follows.

19. Portfolio Currency Return Attribution per currency

We define, for stocks denominated in currency C, the Portfolio Currency Return Attribution as:

$$FX_C^P = \sum_{i \in C} w_i^P \left(r_i^{base} - r_i^l \right) \qquad \text{(A36)}$$

20. Benchmark Currency Return Attribution

We define, for currency C, the Benchmark Currency Return Attribution as:

$$FX_C^B = \sum_{i \in C} w_i^B \left(r_i^{base} - r_i^l \right) \qquad \text{(A37)}$$

21. Active Currency Return Attribution

We define, for currency C, the Active Currency Return Attribution as:

$$FX_C^a = \sum_{i \in C} a_i \left(r_i^{base} - r_i^l \right) = \sum_{i \in C} \left(w_i^P - w_i^B \right) \left(r_i^{Base} - r_i^l \right) \qquad \text{(A38)}$$

In most cases, currencies correspond directly to countries, and so the currency contributions per country or per currency will be identical in the absence of any cash holdings. However, in the case of the Euro, the appropriate Eurozone per country currency attributions will sum to the overall Eurozone currency component. When the portfolio or benchmark contains cash, this is shown as a separate line in '*Agg Perf*

History' whereas it is included in the active currency return attribution. So the per country and per currency contributions will not match up in this situation.

Special Cases

There are two special cases for the Brinson methodology where there may be holdings in a particular group in the portfolio but not in the benchmark (and *vice versa)*. An example might be where the portfolio holds stocks in Canada when the benchmark is 100% US. In such situations the standard formulae are modified to give meaningful results.

a) $W_G^P \neq 0; W_G^B = 0$

In the case where we have a group represented in the portfolio, but not in the benchmark, the usual Brinson terms no longer make sense, as the benchmark return of that group is un-defined. This may be addressed by using the portfolio group return in place of the benchmark group return for all components. With reference to A29, A31 and A33, this leads to:

$$R_G^{Allocation} = W_G^P (R_G^{Pl} - R^{Bl}) \qquad \text{(A39)}$$

$$R_G^{Selection} = R_G^{Interaction} = 0 \qquad \text{(A40)}$$

This is reasonable as it is not possible to determine the value added from stock selection as there are no stocks in that group in the benchmark with which to make the comparison. Therefore Selection and Interaction terms of zero make sense and all the return is accredited to Allocation[11]**.** In this special case we show the portfolio group return relative to the benchmark return in the ***Contributions by Market*** chart for this group.

[11] Please note, however, that in the (even more) special case where a group is added to or deleted from the benchmark (e.g. when Portugal joined the Developed markets), but is always represented in the portfolio, the cumulative returns will mix up benchmark with portfolio sourced group returns

b) $W_G^P = 0; W_G^B \neq 0$

This is a more simple case. Here the Allocation term is unchanged from A29. However, as the portfolio group return is undefined, it makes no sense to have a stock selection component. Hence both the Selection and Interaction terms are set to zero, as in A40.

The remaining parts of this section relate to fund of funds analysis.

22. Fund Return (Base Currency)

For a particular fund, F, we have:

$$R^{Fbase} = \sum_{i \in F} w_i^F r_i^{base}$$ is the Fund return in base currency terms (A41)

where w_i^F are the stock weights in Fund F (which are input during the original fund load).

23. Fund Benchmark Return (Base Currency)

Similarly

$$R^{FBMbase} = \sum_{i \in FBM} w_i^{FBM} r_i^{base}$$ is the Fund Benchmark return in base terms. (A42)

where w_i^{FBM} are the stock weights in Fund Benchmark FBM.

24. Fund Active Return (Base Currency)

$$R^{Fabase} = R^{Fbase} - R^{FBMbase}$$ (A43)

This is the active return of fund F relative to its fund benchmark FBM.

25. Fund Benchmark vs. Benchmark Return (Base Currency)

$$R^{FBMabase} = R^{FBMbase} - R^{Bbase}$$ (A44)

This measures how fund benchmark FBM performed relative to the overall benchmark.

Cumulative versions of A41-44 are also shown in *PerfFund.*

26. Benchmark Allocation Return (Base Currency)

$$R_F^{Allocation} = W_F \left(R^{FBMbase} - R^{Bbase} \right) \tag{A45}$$

This is employed when the benchmark cannot be expressed in terms of Fund Benchmarks. The weights are the absolute weights of the funds in the fund of funds and are defined in section 29.

Summed across all funds, this gives the total Fund Allocation Contribution at time *t*:

$$R^{FundAllocation} = \sum_{\forall F} R_F^{Allocation} \tag{A46}$$

27. Fund Selection Return (Base Currency)

$$R_F^{Selection} = W_F \left(R^{Fbase} - R^{FBMbase} \right) \tag{A47}$$

Selection measures the active return of the fund versus its fund benchmark times its weight in the fund of funds mix. The weights are defined in section 29.

Summed across all funds, this gives the total Fund Selection Contribution at time *t*:

$$R^{FundSelection} = \sum_{\forall F} R_F^{Selection} \tag{A48}$$

28. Total Contribution (Base Currency)

Per fund, this is the sum of $R_F^{Allocation} + R_F^{Selection}$.

When this is summed across all funds it leads to the overall active return of the fund of funds relative to the benchmark.

29. Fund/Model BM/Active Weights

The total weight in each fund W_F and fund benchmark W_{FBM} are supplied by the user. The latter (sometimes called Model BM Weights) are only available if the benchmark is entirely made up of fund benchmarks. A fund benchmark not present in the benchmark (i.e. one that is associated with a fund but which is not part of the benchmark) has a weight of zero. Active weights per fund are simply the difference between W_F and

W_{FBM} (which will just equal the fund weight in cases where the fund benchmark is not present in the benchmark).

30. Model BM Allocation

$$R_F^{BMAllocation} = \left(W_F - W_{FBM}\right)\left(R^{FBMbase} - R^{Bbase}\right) \quad \text{(A49)}$$

This is what is shown in the charts when the benchmark is made up of fund benchmarks. In this case the weights of the fund benchmarks in the benchmark are assumed to be the passive allocation, so any deviation from these via the fund weights is deemed to be an active allocation.

Over all funds, this sums so the total BM Allocation:

$$R^{BMAllocation} = \sum_{\forall F} R_F^{BMAllocation} \quad \text{(A50)}$$

31. Model BM Selection

This is identical to the Fund Selection term above (A47/A48) irrespective of the constitution of the benchmark.

32. Model BM Total Attrib

This is the sum of the Model BM Allocation and the Model BM Selection (A49 + A47). Summed over all funds this leads to the overall active return of the fund of funds versus the benchmark.

Note that there are links to cumulative and reverse cumulative series for most of these items.

Hierarchical Methodology

The hierarchical methodology is based on a nested categorization of returns, the order of which may be selected by the user. The formulae set out here are based on the default hierarchy of currency, market, sector, style and stock. The software permits other hierarchies to be constructed to suit different investment management processes or simply to offer different attribution perspectives. Their derivation follows in a similar way.

In the hierarchical methodology we define stock *i*'s active base currency return at month end *t*, $r_i^{base}(t) - R^{Bbase}(t)$ as being made up of a hierarchy of return components:

$$r_i^{base}(t) - R^{Bbase}(t) = C_i(t) + M_i'(t) + I_i'(t) + S_i'(t) + E_i'(t) \qquad \text{(B1)}$$

These five components are defined below.

1. Currency Component

This is defined as:

$$C_i(t) = (r_i^{base}(t) - R^{Bbase}(t)) - (r_i^{l}(t) - R^{Bl}(t)) \qquad \text{(see '}\textit{Perf Stock Level Data' col Q}\text{)}$$

It is the return needed in order to convert the local active return into the base currency active return. In terms of exchange rates, and local stock returns, it may also be expressed as:

$$C_i(t) = (1 + r_i^{l}(t))\left(\frac{X^{(i)}(t) - X^{(i)}(t-1)}{X^{(i)}(t-1)}\right) - \sum_{\forall j} w_j^{B}(1 + r_j^{l}(t))\left(\frac{X^{(j)}(t) - X^{(j)}(t-1)}{X^{(j)}(t-1)}\right) \qquad \text{(B2)}$$

where,

$r_i^{l}(t)$ = local stock return for stock *i* between month end $t-1$ and month end t.

$X^{(i)}(t)$ = The exchange rate to convert from the base currency of analysis to the local currency of stock *i* at t.

2. Market Component

This is defined as:

$$M_i'(t) = M_i(t) - R^{Bl}(t) \qquad \text{(B3)}$$

where,

$$M_i(t) = \frac{\sum_{\forall f} w_f^B(t) r_f{}^l(t)}{\sum_{\forall f} w_f^B(t)} \text{ for all stocks } f \text{ where:} \quad \text{(B4)}$$

$f \in market(i)$, the market to which stock *i* belongs

$w_f^B(t)$ = the benchmark weight for stock f at the beginning of month *t* (equivalently, the end of month *t-1).*

$r_f{}^l(t)$ = the local currency return for stock f between month end $t-1$ and month end t.

$R^{Bl}(t)$ = the local currency benchmark return between month end $t-1$ and month end t.

Essentially, the market return is the benchmark weighted local currency return for stock *i*'s country, in excess of the local currency benchmark return.

For the remaining formulae we drop the reference to *t*, with the convention that all returns are between month end $t-1$ and month end t, and all weights refer to the beginning of month *t*.

3. Sector Component

In the default hierarchy this is nested below countries (hence these are 'local industries', such as *Swiss Financials*). This is defined as:

$$I_i' = I_i - M_i \quad \text{(B5)}$$

where,

$$I_i = \frac{\sum_{\forall h} w_f^B r^l{}_f}{\sum_{\forall h} w_f^B} \quad \text{(B6)}$$

where the sum is over all stocks $h \in market(i)$ and $\in sector(i)$

4. Style Component

In the default this is defined *within* sectors *within* countries and is therefore the Style at a local industry level. An example would be *Large Value Swiss Financials*.

$$S_i' = S_i - I_i \quad \text{(B7)}$$

where,

$$S_i = \frac{\sum_{\forall m} w_f^B r_f^{\ l}}{\sum_{\forall m} w_f^B} \qquad \text{(B8)}$$

where the sum is over all stocks $m \in market(i)$ and $\in sector(i)$ and $\in Style(i)$

5. Stock Component

The stock specific (equity) return is defined as:

$$E_i' = r_i^{\ l} - S_i . \qquad \text{(B9)}$$

The return of every stock may now be decomposed into a hierarchy of five components. From this, the active base currency return of a portfolio can be written as:

$$R^{abase} = \sum_{\forall i} (w_i^P - w_i^B)(C_i + M_i' + I_i' + S_i' + E_i') \qquad \text{(B10)}$$

This active return can then be broken out as follows:

$R^{abase} =$

$$\sum_{\forall i} (w_i^P - w_i^B) C_i \qquad \textit{Currency return} \qquad \text{(B11a)}$$

$$+ \sum_{\forall i} (w_i^P - w_i^B) M_i' \qquad \textit{Market return} \qquad \text{(B11b)}$$

$$+ \sum_{\forall i} (w_i^P - w_i^B) I_i' \qquad \textit{Sector return} \qquad \text{(B11c)}$$

$$+ \sum_{\forall i} (w_i^P - w_i^B) S_i' \qquad \textit{Style return} \qquad \text{(B11d)}$$

$$+ \sum_{\forall i} (w_i^P - w_i^B) E_i' \qquad \textit{Stock return} \qquad \text{(B11e)}$$

where,

w_i^P = the portfolio weight for stock *i* (at month end *t-1*)

w_i^B = the benchmark weight for stock (*i* at month end *t-1*)

Further categorization is possible, and can be accessed from the pivot table provided in the output and in the data sheets.

Now that we have the contribution of every stock to active return for each hierarchical component, we can aggregate these returns in different ways. One example might be to split the Stock component according to Styles. The return due to stock picking within e.g. Small Value (defined within sector within country) would be:

$$\sum_{i \in SmallValue} (w_i^P - w_i^B) E_i'$$

Alternatively, other components may be studied: e.g. the return due to sector selection, within the US would be:

$$\sum_{i \in US} (w_i^p - w_i^b) I_i' \qquad \text{etc.}$$

Equivalence of 2-level Hierarchical and Brinson methods

When there are just two levels in a Hierarchical analysis (e.g. market and stock), the approach yields results identical to the Brinson components, as follows:

Hierarchical Market Return = Brinson Allocation
Hierarchical Stock Return = Brinson Selection + Brinson Interaction
= Portfolio Weighted Brinson Selection (see A31b)

To see this, consider the Hierarchical components for this case:

$$R^{abase} =$$

$$\sum_{\forall i} (w_i^P - w_i^B) C_i \qquad \textit{Currency return}$$

$$+ \sum_{\forall i} (w_i^P - w_i^B) M_i' \qquad \textit{Market return}$$

$$+ \sum_{\forall i} (w_i^P - w_i^B) E_i' \qquad \textit{Stock return}$$

The Currency term is the same for both cases. From B11(a):

$$\sum_{\forall i} (w_i^P - w_i^B) C_i = \sum_{\forall i} (w_i^P - w_i^B)[(r_i^{base} - R^{Bbase}) - (r_i^l - R^{Bl})]$$

$$\equiv \sum_{\forall i} a_i \, (r_i^{base} - r_i^l) \qquad \text{(B12)}$$

This is equivalent to the Brinson A21, summed over all groups.

The Market Return is**:**

$$\sum_{\forall i} (w_i^P - w_i^B) M_i' = \sum_{\forall i} (w_i^P - w_i^B)[M_i - R^{Bl}] \qquad \text{(B13)}$$

The sum may be partitioned into different markets, G_1, G_2 etc.

$$\sum_{i \in G_1} (w_i^P - w_i^B)[M_i - R^{Bl}] + \sum_{i \in G_2} (w_i^P - w_i^B)[M_i - R^{Bl}] + \cdots \qquad \text{(B14)}$$

Now, within a particular market, say G_1, the market return is the same for all stocks within that group. i.e.:

$$M_i = \frac{\sum_{f \in G_1} w_f^B r_f^{\ l}}{\sum_{f \in G_1} w_f^B} = R_{G_1}^{Bl}$$

This is simply the benchmark (local currency) return of A23. So the first term of B14 reduces to:

$$\sum_{i \in G_1} (w_i^P - w_i^B)[R_{G_1}^{Bl} - R^{Bl}] = (W_{G_1}^P - W_{G_1}^B)(R_{G_1}^{Bl} - R^{Bl}) = R_{G_1}^{Allocation}$$

which is identical to A29, the Brinson allocation contribution. Across all groups this will sum to the overall Allocation contribution as per A30.

The Stock Return for the *two layer* case is defined as:

$$\sum_{\forall i} (w_i^P - w_i^B) E_i' = \sum_{\forall i} (w_i^P - w_i^B)(r_i^{\ l} - M_i)$$

As before, this may be split into different markets, G_1, G_2 etc.

$$\sum_{i \in G_1} (w_i^P - w_i^B)(r_i^{\ l} - M_i) + \sum_{i \in G_2} (w_i^P - w_i^B)(r_i^{\ l} - M_i) + \cdots$$

Recall from above, that within a particular market, say G_1, all $M_i = R_{G_1}^{Bl}$. Therefore, using this, and the results of A23 and A24, the first term above is equivalent to:

$$\sum_{i \in G_1} (w_i^P - w_i^B)(r_i^{\ l} - R_{G_1}^{Bl}) = W_{G_1}^P R_{G_1}^{Pl} - W_{G_1}^P R_{G_1}^{Bl} + W_{G_1}^B R_{G_1}^{Bl} - W_{G_1}^B R_{G_1}^{Bl} = W_{G_1}^P (R_{G_1}^{Pl} - R_{G_1}^{Bl})$$

This is the *portfolio weighted* Brinson Stock Selection Contribution for G_1 as per A31b. This is equivalent to the sum of the *benchmark weighted* Brinson Stock Selection Contribution plus the Brinson Interaction Contribution.

Appendix P3: Multi Period Return Attribution

In all of the attributions that we have discussed, the end result was to decompose the return over a particular month, *t,* into *G* component returns:

$$R\ (t) = \sum_{\forall G} R_G(t)$$

This is the approach favoured by most investment practitioners because it only requires simple addition of the components to get back to the overall return. However, over multiple periods, this can cause difficulties, because overall returns cumulate geometrically, but the cumulative arithmetic components no longer sum to the overall return.

The overall cumulative return over the next *n* months is:

$$C(t+n) = [1+R(t)][1+R(t+1)][1+R(t+2)]\cdots[1+R(t+n)]-1$$

A similar definition *could* be made for the cumulative components, $C_G(t+n)$, i.e.

$$C_G(t+n) = [1+R_G(t)][1+R_G(t+1)][1+R_G(t+2)]\cdots[1+R_G(t+n)]-1$$

However, the sum of the cumulative components across all groups does not add up to give the overall cumulative return, i.e.:

$$C(t+n) \neq \sum_{\forall G}\ C_G(t+n)$$

Here's a simple two group example to illustrate the point. Suppose we had overall portfolio returns of 15% over each of two months. Suppose also that group 1 contributed 10% return in each month, and group 2 the other 5%. Over the whole period the cumulative return is 1.15*1.15-1 = 32.25%. However, the first group component cumulates to 21% and the second to 10.25%, and these add to give 31.25% ≠ 32.25%.

In order to get around this difficulty we use a standard linking methodology proposed by David R. Carino of Frank Russell 1999[12]. We start by defining log returns as:

$$\tilde{R}(t) = \ln(1+R(t))$$

Similarly, on a cumulative basis we can define:

$$\tilde{C}(t+n) = \ln[1+C(t+n)]$$

Log returns have nice properties, in particular, that the per month log returns add up to the cumulative log return: i.e.:

[12] "Combining Attribution Effects over Time" Journal of Performance Measurement, Summer 5-14.

$$\tilde{C}(t+n) = \sum_{k=0}^{n} \tilde{R}(t+k)$$

Using this fact, we can define cumulative returns at the group level that sum to the overall cumulative return, as follows:

$$C'_G(t+n) = \frac{C(t+n)}{\tilde{C}(t+n)} \sum_{k=0}^{n} \frac{\tilde{R}(t+k)}{R(t+k)} R_G(t+k)$$

Essentially, the underlying per month group returns are scaled using factors based on the ratio of log to regular returns for the period of interest. It is easy to show that, using this definition[13]:

$$C(t+n) = \sum_{\forall G} C'_G(t+n)$$

That is, the cumulative components at any point in time sum to give the overall cumulative return at that point, as required. Therefore, this is a neat way of scaling the group level components so that at any time they will add to give the overall cumulative return at that point.

The various cumulative returns reported in the charts and data sheets use this generic method. There are separate calculations for the portfolio, benchmark and active cases, and for local and base currency terms – making 6 varieties in all. These variations are necessary so that the components add to the appropriate cumulative return at that point (e.g. the active return in local currency). It is worth noting however, for the Brinson components, which report local currency contributions for allocation, selection and interaction, that the calculations are set so that the terms across these components plus cash and currency elements add to give the cumulative active base currency return.

[13] The summation over G yields the overall return R(*t+k)* which cancels the same in the denominator. We are left with the sum over time of the log returns, which yields the cumulative log return, which in turn cancels the same in the denominator, leaving *C(t+n)* as required.